ALAMO COLLEGES

NORTHWEST VISTA COLLEGE

Math for Liberal Arts

Math 1332

Spring, 2011 Edition

BULK FOODS

PREPACK MILK CHOCOLATE COVERED PECANS

milk chocolate (sugar,cocoa butter,milk,ch olate liquor,soy lecithin (emulsifier),vanillin (artificial flavor)),partially hydrogenated palm kernel oil,cocoa powder,pecans,confectioner's glaze

packed on: 08.21.10 sell by: 09.11.10 ⊙ 15:25

NET WEIGHT 0.36 lb $/lb 9.99

$ 3.60

0280056 203604 191

Central Market Austin, TX. 78756

Nutrition Facts

Serving Size 9 pieces (40 g)
Servings Per Container Varied

Amount Per Serving	
Calories 240	Calories from Fat 160
	%Daily Value *
Total Fat 1828.0g	2812%
Saturated Fat 40.0g	200%
Trans Fat 0.0g	
Cholesterol 0mg	0%
Sodium 10mg	0%
Total Carbohydrate 19g	6%
Dietary Fiber 2g	8%
Sugars 17g	
Sugar Alcohols 0.0g	
Protein 2g	

Vitamins A 0%	• Vitamins C 0%
Calcium 2%	• Iron 2%

* Percent Daily Values are based on a 2000 calorie diet. Your daily values may be higher or lower depending on your calorie needs:

	Calories:	2000	2500
Total Fat	Less than	65g	80g
Saturated Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium		2400mg	2400mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g

Calories per gram:
Fat 9 • Carbohydrate 4 • Protein 4

Assuming that the net weight of the chocolate-covered pecans is correctly stated at 0.36 pounds, how worried should one be about the total fat content of 1828.0g?

Contents

Northwest Vista College
Course Syllabus
Math 1332 (Liberal Arts Mathematics)

Instructor:

Office Hours:

Office : Phone: **e-mail:**

Course description. Prerequisite: Math 0302 with a grade of C or better, or equivalent. This course is designated for nonmathematics majors and nonscience majors who need three hours of mathematics for degree requirements. The course includes topics from logic, algebra, trigonometry, and probability. Three lecture hours per week.

Text: *Math for Liberal Arts, Math 1332, Spring, 2010 Edition.* Available only at NVC Bookstore

ASK Outcome: This course will provide the student with sufficient mathematical knowledge for effective living.

Academic Integrity: Please go to http://www.accd.edu/nvc/students/learning/acadinteg/default.htm and read the complete set of policies and procedures regarding academic integrity.

Attendance: Preparation for class, daily attendance, and participation in class are required for a passing grade. Attendance is absolutely mandatory. If a student does miss a class, he or she is responsible to arrange to make up any homework missed. To be successful in college courses, students are expected to attend class on a regular basis. Students need to be aware that they may be dropped by the instructor after census date for lack of progress, which is often caused by lack of attendance.
(http://www.accd.edu/nvc/students/catalog/viewpage.asp?id=125&c=29).

Assessment: Course grade determination will include at least three, one-hour exams and a comprehensive final exam. Writing assignments, homework, quizzes, projects, and class participation may also be considered.

Course objectives. With emphasis on the **bolded** items, the student will be able to:

(a) exhibit an understanding of elementary, intuitive set theory, including the concepts of subset, union, intersection, and cardinal number

(b) develop symbolic truth tables

(c) change numbers from base ten to other bases and vice versa

(d) exhibit facility with solving problems with the calculator, including integer problems, decimal problems, and percent problems

(e) convert within the metric system and between the metric system and the U.S. customary system

(f) solve elementary linear programming problems

(g) exhibit and understanding of elementary probability

(h) compute the mean, median, mode and standard deviation of a distribution

(i) use the normal curve to solve elementary problems in statistics

(j) calculate simple and compound interest

(k) find the sum of arithmetic and geometric series

Course overview. The purpose of this course is to provide the student with an understanding of practical problem-solving methods and mathematical reasoning and to instill in the student an appreciation for mathematics.

Materials: notebook and scientific calculator recommended.

Performance measurements. There will be two or three major tests in addition to the final exam. The tests will be worth 90% of the grade. The other 10% will be based on homework, quizzes, and class participation.

Math Advocacy Center: The Advocacy Center is available to help students with difficult concepts, study skills, and test-taking strategies. One-on-one tutoring is available in 30 minute sessions either by appointment or walk-in basis in JH 308.

ADA: Students who need services for disabilities defined by Section 504 of the Rehabilitation Act of 1973 or the Americans with Disabilities Act of 1990 should contact the Access Department by email at nvc-access@mail.accd.edu, or by phone at 486-4466. Please feel free to discuss any other accommodations that you might need with your instructor.

Methods of instruction. After a problem has been posed to the class, discussion will follow. Lectures will be given when needed.

The value of integrity: Northwest Vista College values integrity; therefore cheating will not be tolerated. Please go to http://www.accd.edu/nvc/docs/catalog/13handbook/conduct.html and read the complete set of new policies and procedures regarding academic integrity.

The following are tentatively among the topics to be discussed. Emphasis will be on solving word problems.

Week 1 set theory, sets of real numbers

Week 2 truth tables

Week 3 number bases

Week 4 word problems

Week 5 the metric system

Week 6 linear and quadratic equation

Week 7 graphing lines

Week 8 systems of linear equations

Week 9 linear programming

Week 10 the Pythagorean Theorem, perimeter, area, volume

Week 11 introduction to probability and statistics

Week 12 mean, median, mode, standard deviation

Week 13 using the normal curve

Week 14 simple and compound interest

Week 15 arithmetic and geometric series

p 3

Pathways to Learning

Directions: Rate each statement as follows:

rarely	sometimes	often	almost always
1	2	3	4

Write the number of your response (1-4) on the line next to the statement number.

1. _____ I enjoy physical activities.
2. _____ I am uncomfortable sitting still.
3. _____ I prefer to learn through doing rather than listening.
4. _____ I tend to move my legs or hands when I'm sitting.
5. _____ I enjoy working with my hands.
6. _____ I like to pace when I'm thinking or studying.

_______ **TOTAL for Bodily-Kinesthetic**

7. _____ I use maps easily.
8. _____ I draw pictures or diagrams when explaining ideas.
9. _____ I can assemble items easily from diagrams.
10. ____ I enjoy drawing or photography.
11. ____ I do not like to read long paragraphs.
12. ____ I prefer a drawn map over written directions.

_______ **TOTAL for Visual-Spatial**

13. ____ I enjoy telling stories.
14. ____ I like to write.
15. ____ I like to read.
16. ____ I express myself clearly.
17. ____ I am good at negotiating.
18. ____ I like to discuss topics that interest me.

________ **TOTAL for Verbal-Linguistic**

19. ____ I liked math in school.
20. ____ I like science.
21. ____ I problem-solve well.
22. ____ I question why things happen or how things work.
23. ____ I enjoy planning or designing something new.
24. ____ I am able to fix things.

________ **TOTAL for Logical-Mathematical**

25. ____ I listen to music.
26. ____ I move my fingers or feet when I hear music.
27. ____ I have good rhythm.
28. ____ I like to sing along with music.
29. ____ People have said I have musical talent.
30. ____ I like to express my ideas through music.

_______ **TOTAL for Musical**

31. ____ I like doing a project with other people.
32. ____ People come to me to help them settle conflicts.
33. ____ I like to spend time with friends.
34. ____ I am good at understanding people.
35. ____ I am good at making people feel comfortable.
36. ____ I enjoy helping others.

_______ **TOTAL for Interpersonal**

37. ____ I need quiet time to think.
38. ____ When I need to make a decision, I prefer to think about it before I talk about it.
39. ____ I am interested in self-improvement.
40. ____ I understand my thoughts, feelings, and behavior.
41. ____ I know what I want out of life.
42. ____ I prefer to work on projects alone.

_______ **TOTAL for Intrapersonal**

43. ____ I enjoy nature whenever possible.
44. ____ I would enjoy a career involving nature.
45. ____ I enjoy studying plants, animals, forests, or oceans.
46. ____ I prefer to be outside whenever possible.
47. ____ When I was a child I liked bugs, plants, and leaves.
48. ____ When I experience stress I want to be out in nature.

_______ **TOTAL for Naturalistic**

Dev. by Joyce Bishop, reference *Keys to Success* 2ed, Prentice-Hall; based upon Howard Gardner's theory of Multiple Intelligences;

Pathways to Learning - Multiple Intelligences

SKILLS

Verbal / Linguistic
- Analyzing own use of language
- Remembering terms easily
- Explaining, teaching, learning, & using humor
- Understanding syntax and meaning of words
- Convincing someone to do something

Musical / Rhythmic
- Sensing tonal qualities
- Creating or enjoying melodies and rhythms
- Being sensitive to sounds and rhythms
- Using "schemas" to hear music
- Understanding the structure of music

Logical / Mathematical
- Recognizing abstract patterns
- Reasoning inductively and deductively
- Discerning relationships and connections
- Performing complex calculations
- Reasoning scientifically

Visual / Spatial
- Perceiving and forming objects accurately
- Recognizing relationships between objects
- Representing something graphically
- Manipulating images
- Finding one's way in space

Bodily / Kinesthetic
- Connecting mind and body
- Controlling movement
- Improving body functions
- Expanding body awareness to all senses
- Coordinating body movement

Intrapersonal
- Evaluating own thinking
- Being aware of and expressing feelings
- Understanding self in relationship to others
- Thinking and reasoning on higher levels

Interpersonal
- Seeing things from others' perspectives
- Cooperating within a group
- Communicating verbally and nonverbally
- Creating and maintaining relationships

LEARNING TECHNIQUES

Verbal / Linguistic
- Read text and highlight no more than 10%
- Rewrite notes
- Outline chapters
- Teach someone else
- Recite information or write scripts/debates

Musical / Rhythmic
- Create rhythms out of words
- Beat out rhythms with hand or stick
- Play instrumental music/ write raps
- Put new material to songs you already know
- Take music breaks

Logical / Mathematical
- Organized material logically
- Explain it sequentially to someone
- Develop systems and find patterns
- Write outlines and develop charts and graphs
- Analyze information

Visual / Spatial
- Develop graphic organizers for new material
- Draw mind maps
- Develop charts, and graphs
- Use color in notes to organize
- Visualize material (method of loci)

Bodily / Kinesthetic
- Move or tap while you learn; pace and recite
- Use "method of loci" or manipulatives
- Move fingers under words while reading
- Create "living sculptures"
- Act out scripts of material, design games

Intrapersonal
- Reflect on personal meaning of information
- Visualize information / keep a journal
- Study in quiet setting
- Imagine experiments

Interpersonal
- Study in a group
- Discuss information
- Use flash cards with others
- Teach someone else

Adapted by Joyce Bishop (714)895-8304 from © David Lazear, *Seven Pathways of Learning*, 1994.

Personality Spectrum

STEP 1. Rank order all 4 responses to each question from <u>most like you (4)</u>, to <u>least like you (1)</u>. Place a 1, 2, 3, or 4 in each gray box next to the responses.

1. I like instructors who
   - a. tell me exactly what is expected of me.
   - b. make learning active and exciting.
   - c. maintain a safe and supportive classroom.
   - d. challenge me to think at higher levels.

2. I learn best when the material is
   - a. well organized.
   - b. something I can do hands-on.
   - c. about understanding and improving the human condition.
   - d. intellectually challenging.

3. A high priority in my life is to
   - a. keep my commitments.
   - b. experience as much of life as possible.
   - c. make a difference in the lives of others.
   - d. understand how things work.

4. Other people think of me as
   - a. dependable and loyal.
   - b. dynamic and creative.
   - c. caring and honest.
   - d. intelligent and inventive.

5. When I experience stress I would most likely
   - a. do something to help me feel more in control of my life.
   - b. do something physical and daring.
   - c. talk with a friend.
   - d. go off by myself and think about my situation.

6. The greatest flaw someone can have is to be
   - a. irresponsible.
   - b. unwilling to try new things.
   - c. selfish and unkind to others.
   - d. an illogical thinker.

7. My vacations could be best described as
   - a. traditional.
   - b. adventuresome.
   - c. pleasing to others.
   - d. a new learning experience.

8. One word that best describes me is
   - a. sensible.
   - b. spontaneous.
   - c. giving.
   - d. analytical.

STEP 2. Add up the total points for each column.

TOTAL	TOTAL	TOTAL	TOTAL
Organizer	Adventurer	Giver	Thinker

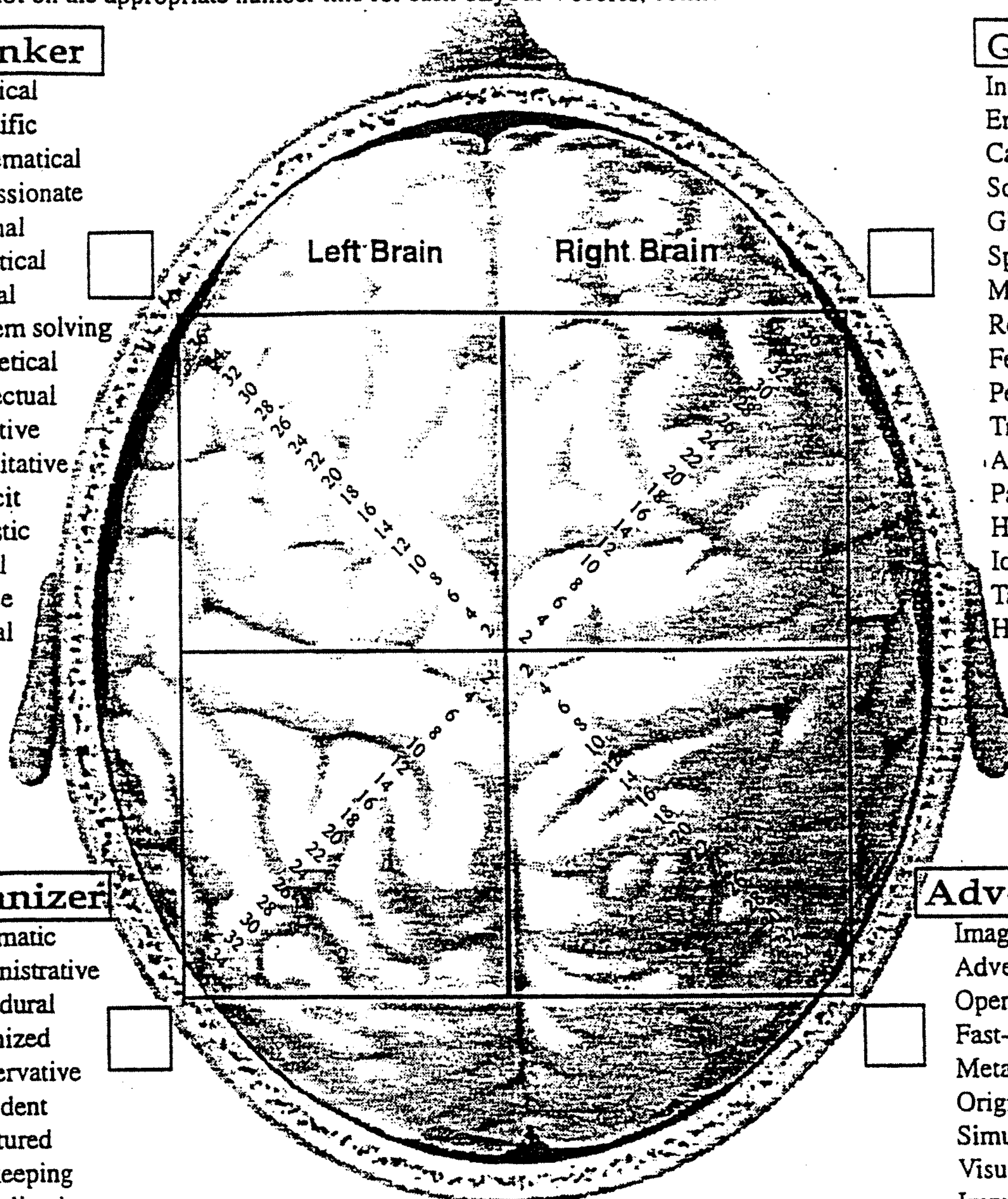
STEP 3. Plot these numbers on the brain diagram on the next page.

Reference *Keys to Success* 2ed, Prentice-Hall; Joyce Bishop, Ph.D. (1997) 714-895-8304, jbishop@gwc.cccd.edu

Personality Spectrum --Thinking Preferences & Learning Styles

reference *Keys to Success* 2ed. Prentice-Hall; Joyce Bishop, Ph.D., (714)846-4706; jbishop@gwc.cccd.edu

Place a dot on the appropriate number line for each of your 4 scores, connect the dots and color each polygram.



Thinker

Technical
Scientific
Mathematical
Dispassionate
Rational
Analytical
Logical
Problem solving
Theoretical
Intellectual
Objective
Quantitative
Explicit
Realistic
Literal
Precise
Formal

Giver

Interpersonal
Emotional
Caring
Sociable
Giving
Spiritual
Musical
Romantic
Feeling
Peacemaker
Trusting
Adaptable
Passionate
Harmonious
Idealistic
Talkative
Honest

Organizer

Systematic
Administrative
Procedural
Organized
Conservative
Confident
Structured
Safekeeping
Disciplined
Practical
Sequential
Predictable
Detailed
Tactical
Controlled
Dependable
Planning

Adventurer

Imaginative
Adventuresome
Open-minded
Fast-paced
Metaphoric
Original
Simultaneous
Visual
Impulsive
Experimental
Risking
Divergent
Artistic
Spatial
Skillful
Competitive
Active

Pathways to Learning

Write your 8 Multiple Intelligences in the columns below according to your total scores.

Scores 20-24 = Highly Developed	Scores 14-19 = Moderately Developed	Scores below 14 = Underdeveloped

Personality Spectrum

Thinker

Personal strengths - You feel best about yourself when solving problems. You love to develop models and systems. You have an abstract and analytical way of thinking. You love to explore ideas. You dislike unfairness and wastefulness. You are global by nature, always seeking universal truth.
Work/school - You work best when assigned projects which require analytical thinking and problem-solving. You are inspired by futuristic ideas and potentials. You need the freedom to go beyond the established rules. You feel appreciate when praised for your ingenuity. You dislike repetitive tasks.
Relationships - You thrive in relationships that recognize your need for independence, and private time to think and read. Stress can come from the fear of appearing foolish. You want others to accept that you feel deeply even though you may not often express it.
Learning - You like quiet time to reflect on new information. Learning through problems-solving and designing new ways of approaching issues is most interesting to you. You may find it effective to convert material you need to learn into logical charts and graphs.

Organizer

Personal strengths - You value order and cherish the traditions of home and family. You support social structures. Generous and parental by nature, you show you care by making sure everyone does the "right thing". You would never disregard responsibility. You have a strong sense of history, culture, and dignity. You value order and predictability. You dislike disobedience or nonconformity. You value loyalty and obligation.
Work/school - You enjoy work that requires detailed planning and follow-through. You prefer to have tasks defined in clear and concrete terms. You need a well-structured, stable environment, free from abrupt changes. You feel appreciated when you are praised for neatness, organization and efficiency. You like frequent feedback so you know you are on the right track.
Relationships - You do best in relationships that provide for your need of security, stability, and structure. You appreciate it when dates that are important to you are remembered by others.
Learning - You must have organization to the material and know the overall plan and what will be required of you. Depending on your most developed Multiple Intelligences, organizing the material could include any of the following: highlighting key terms in text, rewriting and organizing notes from class or the text, making flash cards.

Giver

Personal strengths - You value honesty and authenticity above all else. You enjoy close relationships with those you love and there is a strong spirituality in your nature. Making a difference in the world is important to you, and you enjoy cultivating potential in yourself and others. You are a person of peace. You are a natural romantic. You dislike hypocrisy and deception.
Work/school - You function best in a warm, harmonious working environment with the possibility of interacting with openness and honesty. You prefer to avoid conflict and hostility. You thrive when your creative approach to your work is appreciated and praised.
Relationships - You thrive in relationships that include warm, intimate talks. You feel closer to people when they express their feelings and are open and responsive. You think romance, touch, and appreciation are necessary for survival. You blossom when others express a loving commitment to you and you are able to contribute to the relationship.
Learning - You enjoy studying with others and also helping them learn. Study groups are very effective for you to remember difficult information.

Adventurer

Personal strengths - Your strength is skillfulness. You need freedom to pursue your zest for life and your desire to test the limits. You take pride in being highly skilled in a variety of fields. Adventure is your middle name. A hands-on approach to problems solving is important to you. You need variety and waiting is like "emotional death". You live in the here and now. It is your impulsiveness that drives everything you do. You dislike rigidness and authority.
Work/school - You function best in a work environment that is action -packed with a hands-on approach. You appreciate the opportunity to be skillful and adventurous, and to use your natural ability as a negotiator. You like freedom on the job so you can perform in nontraditional ways and in your own style. Keeping a good sense of humor and avoiding boredom on the job is important to you. You feel appreciated when your performance and skills are acknowledged.
Relationships - You function best in relationships that recognize your need for freedom. You thrive on spontaneous playfulness and excitement.
Learning - You learn exciting and stimulating information easiest so pick classes and instructors carefully. Study with fun people in a variety of ways and places. Keep on the move. Develop games and puzzles to help memorize terminology.

The Geometry of Music

A composer has taken equations from string theory to explain why Bach and bebop aren't so different

By MICHAEL D. LEMONICK

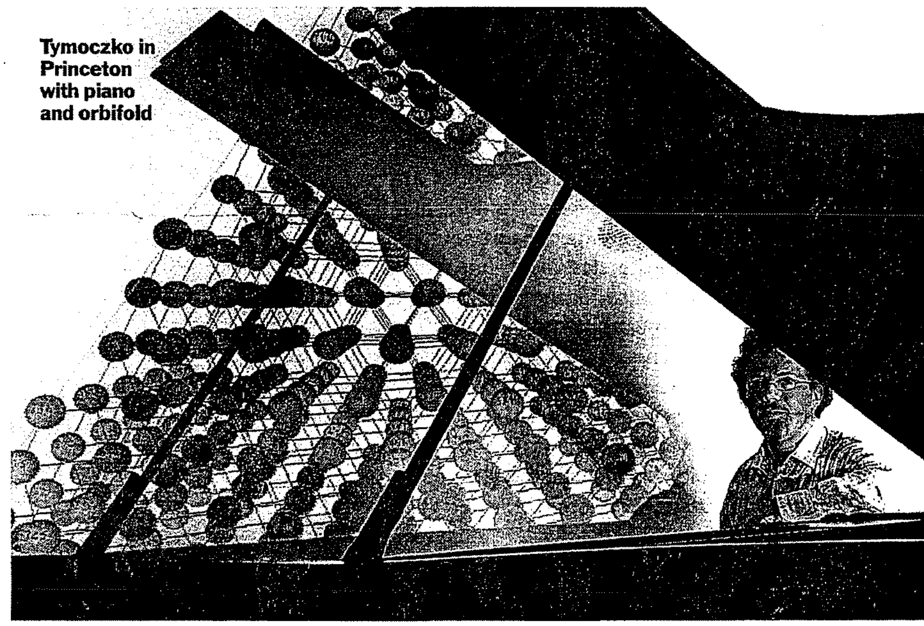
WHEN YOU FIRST HEAR THEM, A Gregorian chant, a Debussy prelude and a John Coltrane improvisation might seem to have almost nothing in common—except that they all include chord progressions and something you could plausibly call a melody. But music theorists have long known that there's something else that ties these disparate musical forms together. The composers of these and virtually every other style of Western music over the past millennium tend to draw from a tiny fraction of the set of all possible chords. And their chord progressions tend to be efficient, changing as few notes, by as little as possible, from one chord to the next.

Exactly how one style relates to another, however, has remained a mystery—except over one brief stretch of musical history. That, says Princeton University composer Dmitri Tymoczko, "is why, no matter where you go to school, you learn almost exclusively about classical music from about 1700 to 1900. It's kind of ridiculous."

But Tymoczko may have changed all that. Borrowing some of the mathematics that string theorists invented to plumb the secrets of the physical universe, he has found a way to represent the universe of all possible musical chords in graphic form. "He's not the first to try," says Yale music theorist Richard Cohn. "But he's the first to come up with a compelling answer."

Tymoczko's answer, which led last summer to the first paper on music theory ever published in the journal *Science,* is that the cosmos of chords consists of weird, multidimensional spaces, known as orbifolds, that turn back on themselves with a twist, like the Möbius strips math teachers love to trot out to prove to students that a two-dimensional figure can have only one side. Indeed, the simplest chords, which consist of just two notes, live on an actual Möbius strip. Three-note chords reside in spaces that look like prisms—except that opposing faces connect to each other. And more complex chords inhabit spaces that are as hard to visualize as the multidimensional universes of string theory.

But if you go to Tymoczko's website (*music.princeton.edu/~dmitri*), you can see exactly what he's getting at by looking at movies he has created to represent tunes by Chopin and, of all things, Deep Purple. In both cases, as the music progresses, one chord after another lights up in patterns that occupy a surprisingly small stretch of musical real estate. According to Tymoczko, most pieces of chord-based music tend to do the same, although they may live in a different part of the orbifold space. Indeed, any conceivable chord lies somewhere in that space, although most of them would sound screechingly harsh to human ears.



Tymoczko in Princeton with piano and orbifold

PETER MURPHY FOR TIME

The discovery is useful for at least a couple of reasons, says Tymoczko. "One is that composers have been exploring the geometrical structure of these maps since the beginning of Western music without really knowing what they were doing." It's as though you figured out your way around a city like Boston, for example, without realizing that some of your routes intersect. "If someone then showed you a map," he says, "you might say, 'Wow, I didn't realize the Safeway was close to the disco.' We can now go back and look at hundreds of years of this intuitive musical pathmaking and realize that there are some very simple principles that describe the process."

That's likely to help both scholars and teachers, he argues. By showing how compositions of various styles move through his orbifold spaces, says Tymoczko, you can see how different styles of Western music relate to each other and evolve. Tymoczko's maps can also be an aid to composers, says Cohn. Most have a favorite corner in orbifold space, a set of related chord types that they tend to explore over and over in different ways. Venturing into a different part of space can be tough; you have to learn your way around a whole new auditory neighborhood. You can do that intuitively by wandering around

> **"Composers have been exploring these maps without really knowing."**
>
> —DMITRI TYMOCZKO

and seeing where you get to. But with the maps, you can plot a route that you know in advance will make some sort of sense.

That doesn't mean you can program a computer with Tymoczko's orbifold maps and have it spit out beautiful compositions. "I don't want to sell these maps as the royal road to composition," he warns. "They don't substitute for the hard work of learning how to move notes around." But they can help show when a new idea is promising and when it will probably lead to a dead end. "They might make an O.K. composer good," says Tymoczko, "but they won't make a good composer great." ■

Name____

Put a check mark to indicate which of the numbers in the top row are divisors of the numbers in the left column.

	2	3	4	5	6	7	8	9	10
39									
51									
57									
87									
91									
93									
98									
126									
246									
264									
720									
1001									

Put a check mark below the fractions that are in lowest terms, if any. In the second row, first list the numbers that one might *try* to use to reduce them, then do it!

$\frac{87}{93}$	$\frac{84}{98}$	$\frac{51}{57}$	$\frac{63}{77}$	$\frac{439}{455}$

Problem	Answer		Answer
5^2		35(35)	
15^2		34(36)	
25^2		33(37)	
35^2		32(38)	
45^2		31(39)	
55^2		72(78)	
65^2		10003(10007)	
75^2		61(59)	
85^2		89(91)	
95^2		101(99)	

A2

1. Divisibility Tests

Divisor	Test for divisibility
2	The unit's digit is even: 0, 2, 4, 6, or 8.
3	An integer is divisible by 3 if and only if its digital sum* is divisible by 3.
4	An integer is divisible by 4 iff the number formed by its last 2 digits is divisible by 4.
5	An integer is divisible by 5 iff its last digit is 0 or 5.
6	An integer is divisible by 6 iff it is divisible by both 2 and 3.
7	An integer is divisible by 7 iff 7 divides the difference between twice the unit's digit and the number formed by dropping the unit's digit. Illustration: Does 7 divide 819? 81 – 2(9) = 63. Since 7 divides 63, it also divides 819. (As an aside, note that 7 *does* divide 0.)
8	An integer is divisible by 8 iff the number formed by its last 3 digits is divisible by 8
9	number is divisible by 9 iff its digital sum* is divisible by 9
10	An integer is divisible by 10 iff its unit's digit is 0
11, 13, 17, 19…	Divisibility tests exist for other numbers as well, but they are somewhat complicated.

* The digital sum of an integer is the sum of its digits, with the process being repeated until a single digit is obtained. For example, find the digital sum of 834.
Solution: 8 + 3 + 4 = 15; 1 + 5 = 6. The digital sum is 6.

Put a check mark to indicate which of the numbers in the top row are divisors of the numbers in the left column.

	2	3	4	5	6	7	8	9	10
39									
51									
57									
87									
91									
98									
126									
246									
264									
720									
1001									

Fun fact. A positive integer that is not prime has a divisor less than or equal to its square root. Hence, any integer from 2 to 120 is prime or divisible by 2, 3, 5, or 7. (Note: The number 1 is not considered to be prime *or* composite.)

Determine at least one proper divisor of each of the following or write "prime."

39	
51	
57	
59	
61	
67	
87	
89	
91	
97	
101	
103	
107	
109	
111	
113	
115	
117	
119	

2. Fun exercise. Pick an integer with four or five digits. Form a different number with those same digits. Calculate the difference between the two numbers. Circle one of the nonzero digits in the answer. Tell me what digits you did *not* circle, and I'll tell you the one you *did* circle. Illustration. Take 5743. Another number that uses these digits is 7435. Calculate 7435 – 5743 = 1692. Circle one of the digits: 1, 6, 9, 2 and tell me the other three.

3. Fun fact that explains #2. The difference between two integers that consist of the same set of digits is *always* divisible by 9. Now, recall the divisibility test for 9. Do you see how the trick works?

4. Here's a little number theory. In this discussion, all of the letters represent integers.

Definition. The symbol "$d|n$" is read "d divides n," which means that there is an integer m such that $n = dm$.

Theorem. If d divides both a and b, then d divides $ax + by$ for all integers x and y; in particular, d must divide $x - y$. (The converse of this theorem is also true.)

Corollary. If the fraction $\frac{a}{b}$ can be reduced by dividing both a and b by d, then d must also divide the difference $a - b$ (or $b - a$).

This result is sometimes useful in determining the possible common divisors of a fraction's numerator and denominator.

Put a check mark below the fractions that are in lowest terms, if any. In the second row, first list the numbers that one might *try* to use to reduce them, then do it!

$\frac{27}{33}$	$\frac{84}{91}$	$\frac{51}{54}$	$\frac{48}{57}$	$\frac{3459}{3467}$

5. What numbers seem to come next in the following sequence? Find at least *two* different ways of arriving at the same conclusion.

2, 6, 12, 20, 30, 42, 56, 72, 90, _______, _______, _______

6. Study the squares given in the two columns on the left. Does it seem that squaring an integer whose unit's digit is 5 results in a number than ends in 25? Do you think that this pattern continues indefinitely? Can you prove it? On the right side of the table, the "answers" have been split into two columns. What pattern do you discern in the second column from the right?

Problem	Answer	Classroom tip This can be		First digits of answer:	Last two digits of answer:
5^2	25	used to come	5^2	n/a	25
15^2	225	up with a point	15^2	2	25
25^2	625	on a graph to	25^2	6	25
35^2	1225	illustrate a	35^2	12	25
45^2	2025	concept. E.g.,	45^2	20	25
55^2	3025	the graph of	55^2	30	25
65^2	4225	$y = x^2$ contains	65^2	42	25
75^2	5625	the point	75^2	56	25
85^2	7225	(3.5, 12.25),	85^2	72	25

95^2	9025	which you can	95^2	90	25
105^2	11025	come up with	105^2	110	25
115^2	13225	instantly if you	115^2	132	25
125^2	15625	know the trick!	125^2	156	25

7. In your own words, describe how you would square a number that "ends in 5."

8. It turns out that squaring a number that ends in 5 is a special case of a more general fact: it is not necessary that the unit's digits end in 5; it is enough that the two numbers are identical except possibly for their unit's digits, but *the sum of their unit's digits must be 10.* Illustrations.

25(25)	6	25
24(26)	6	24
23(27)	6	21
22(28)	6	16
21(29)	6	09
62(68)	42	16
83(87)	72	21
94(96)	90	24
103(107)	110	21
10002(10008)	1001000	16
100004(100006)		
1002(1008)		
100003(100007)		

9. Using algebra in arithmetic. Recall that $(x + y)(x - y) = x^2 - y^2$

This fact makes it easy to do some special calculations, particularly when $y = 1$ and we know, or can easily calculate, the value of x^2.

Illustration. $51(49) = 50^2 - 1 = 2500 - 1 = 2499$.

In learning the multiplication facts, if one knows the squares, then he or she can calculate the answer to a problem whose factors differ by 2. Illustration: $9(7) = ?$ Since 9 and 7 are "two apart," their product is 1 less than the square of the "number in the middle," 8.

A6

Name____ Posttest

Put a check mark to indicate which of the numbers in the top row are divisors of the numbers in the left column.

	2	3	4	5	6	7	8	9	10
39									
51									
57									
87									
91									
93									
98									
126									
246									
264									
720									
1001									

Put a check mark below the fractions that are in lowest terms, if any. In the second row, first list the numbers that one might *try* to use to reduce them, then do it!

$\frac{87}{93}$	$\frac{84}{98}$	$\frac{51}{57}$	$\frac{63}{77}$	$\frac{439}{455}$

Problem	Answer		Answer
5^2		35(35)	
15^2		34(36)	
25^2		33(37)	
35^2		32(38)	
45^2		31(39)	
55^2		72(78)	
65^2		10003(10007)	
75^2		61(59)	
85^2		89(91)	
95^2		101(99)	

The Real Numbers

The Natural Numbers consist of the set: $N = \{1, 2, 3 \ldots\}$. Natural numbers are also called counting numbers.

Integers consist of the set: $J = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$.

Rational numbers consist of the set of all fractions, where the numerators and denominators are integers, with the denominator not zero:

$Q = \left\{ \frac{a}{b} : a, b \in J, b \neq 0 \right\}$

The real numbers, R, consist of all those numbers represented on the real number line from minus infinity to infinity.

Facts:

1. All natural numbers are integers.

2. All integers are rational numbers. To see this, note that $7 = \frac{7}{1}$

3. All rational numbers are real numbers.

4. Rational numbers are characterized by the fact that their decimal representations are either terminating or repeating.

5. Real numbers are either rational or not rational. Those that are not rational are called *irrational.* Irrational numbers are characterized by the fact that their decimal representations are neither terminating nor repeating. The irrational numbers include all radicals that do not "come out even," such as $\sqrt{2}$, $\sqrt[3]{11}$, etc. Included among the irrational numbers are numbers like π, whose decimal representations are infinite but do not repeat. For practical purposes, we use "rational approximations" to estimate the irrational numbers that arise in everyday life. For example, we often use 3.14 or 3.1416 or $\frac{22}{7}$ for π, because they are relatively *close* to π, whereas π is actually irrational, and has in fact been calculated to millions of decimal places, beginning with:

   3.14159265358979323846264338327950288419716939937510582O…

6. All real numbers are either positive or negative, except zero, which is neither.

7. If two numbers are equal, then whatever is true about one is true about the other. For example, since $\frac{14}{2} = 7$, and 7 is a natural number, then $\frac{14}{2}$ is also a natural number.

Illustrations:

8. Changing a terminating decimal to a fraction: $0.45 = \frac{45}{100} = \frac{9}{20}$
9. Changing a repeating decimal to a fraction: $0.\overline{37} = \frac{37}{99}$
10. Changing a repeating decimal to a fraction when the repetend is preceded by one or more digits that do *not* repeat: the denominator contains one 9 for each digit that repeats followed by one zero for each digit that does *not* repeat. The numerator consists of the number formed by the first group of digits that include the repeating sequence, minus the number formed by the digits that do not repeat. For example, consider 0.54747474747...

$$0.5\overline{47} = \frac{547-5}{990} = \frac{542}{990} = \frac{271}{495}$$

Exercise. For each of the following numbers, check all that apply.

	Natural Number	Integer	Rational Number	Irrational Number	Real number	Not a real number
146						
-12						
$\sqrt{7}$						
$\sqrt{9}$						
$\frac{2}{3}$						
$\frac{105}{7}$						
$-5\pi + 5\pi$						
$\frac{16\sqrt{9}}{-48}$						
$\sqrt{-16}$						
$\frac{2\pi}{3\pi}$						
$\frac{6\pi}{3\pi}$						
$\sqrt[3]{-8}$						

Course: 0301-006

Instructor: GITTINGER D

Report Print Date: 4/16/2007

If six teachers can mark 36 papers in an hour and a half, how many papers can eight teachers mark in one hour?

Student	Number of Visits	Total Time (minutes)
	3	92.00
	27	1,906.00
	2	110.00
	10	590.00
	1	9.00
	7	623.00
	5	370.00
	6	454.00
	2	118.00
	9	490.00

A9.1

Assumptions	Illustration
If the number of workers is constant, then the amount of work that they can do is directly proportional to the amount of time they work.	If the number of workers is constant, but time is doubled, they can do twice as much work. If the number of workers is constant, but the work is doubled, it will take twice as much time. If the number of workers is constant, but time is cut in half, they can do only half as much work. If the amount of time is constant, but the number of workers is cut in half, they can do only half as much work.
If the time to complete the work is constant, then the number of workers required is directly proportional to the amount of work.	If time is constant, but the amount of work doubles, it will take twice as many workers. If time is constant, but the number of workers doubles, they can do twice as much work. If time is constant, but the amount of work is cut in half, it will take only half as many workers. If time is constant, but the number of workers is cut in half, they can do only half as much work.
If the amount of work is constant, then the number of workers required is inversely proportional to the time given.	If the amount of work is constant, but the number of workers is doubled, the work will be done in half the time. If the amount of work is constant, but the amount of time is doubled, it will take half as many workers. If the amount of work is constant, but the number of workers is cut in half, then the work will be done in twice the time. If the amount of work is constant, but the time is cut in half, it will take twice as many workers.

Goal: To solve problems involving direct and inverse proportion (or variation).
Illustration. If 15 pipe layers can lay 4 miles of pipe in 6 months, how many pipe layers are required to lay 32 miles of pipe in 12 months? How many miles of pipe can 30 pipe layers lay in 24 months? How long will it take 10 pipe layers to lay 64 miles of pipe?

Pipe layers (*P*)	Miles of pipe (*W*)	Time (*T*)
15	4	6 months
?	32	12 months
30	?	24 months
10	64	?

Assumptions	Equation
The number of workers required to accomplish a given amount of work in a given amount of time is *directly* proportional to the amount of work, but *inversely* proportional to the time allotted for the work to be done. For example, doubling the amount of work doubles the number of workers required, but doubling the time cuts the number of workers required in half.	If *P* workers can do *W* units of work in *T* units of time, then there is a number, *K*, called the constant of proportionality such that: $P = \frac{W}{T}K$

In the original illustration, we are given that *P* = 15 pipe layers, *W* = 4 miles of pipe, and *T* = 6 months. We substitute (15, 4, 6) for (P, W, T) in the formula

$$P = \frac{W}{T}K$$

and solve for *K*, thus:

$$15 = \frac{4}{6}K$$

6(15) = 4*K*

90 = 4*K*

22.5 = *K*

A9.3

In the formula $P = \frac{W}{T}K$, we replace *K* with 22.5: $P = \frac{W}{T} \cdot 22.5 = \frac{22.5W}{T}$
Thus, we have the formula: $P = \frac{2.25W}{T}$, which we use to work the original problems.

1. How many pipe layers are required to lay 32 miles of pipe in 12 months?
   Solution: In the formula for *P*, replace (*W, T*) with (32, 12):

   $P = \frac{22.5(32)}{12}$ and solve for *P*. (Answer: *P* = 60.)

2. How many miles of pipe can 30 pipe layers lay in 24 months?
   Solution: In the formula for *P*, replace (*P, T*) with (30, 24):

   $30 = \frac{22.5W}{24}$ and solve for *W*. (Answer: *W* = 32.)

3. How long will it take 10 pipe layers to lay 64 miles of pipe?

   Solution: In the formula for *P*, replace (*P, W*) with (10, 64):

   $10 = \frac{22.5(64)}{T}$ and solve for *T*. (Answer: *T* = 144.)

Another approach

Assumptions	Equation
The amount of work a given number of workers can accomplish is directly proportional *both* to the number of workers *and* to the time allowed. For example, doubling *either* the number of workers *or* the time doubles the work a given number of workers can do.	If *P* workers can do *W* units of work in *T* units of time, then there is a number, *L*, called the constant of proportionality such that: $W = PTL$
The time it takes a given number of workers to accomplish a given amount of work is *directly* proportional to the amount of work but *inversely* proportional to the number of workers.	If *P* workers can do *W* units of work in *T* units of time, then there is a number, *L*, called the constant of proportionality such that: $T = \frac{W}{P}L$

a. Show that the original set of problems can be solved by beginning with either of the other two equations in the table above, namely

$W = PTK$ or $T = \frac{W}{P}L$

b. Show that the constant, K, in the formula for W is equal to the reciprocal of the L in the formula for T.

Summary of calculations

1. Suppose that A workers can do B units of work in C units of time.

Questions.

1. How many workers, X, are required to accomplish Y units of work in Z units of time?
2. How many units of work, Y, can X workers accomplish in Z units of time?
3. How many units of time, Z, will it take X workers to accomplish Y units of work?

In tabular form:

Number of workers	Units of work	Units of time
A	B	C
X	Y	Z

Formulas:

$$X = A \times \frac{Y}{B} \div \frac{Z}{C} = A \times \frac{Y}{B} \times \frac{C}{Z} = \frac{AYC}{BZ}$$

$$Y = B \times \frac{X}{A} \times \frac{Z}{C} = \frac{BXZ}{AC}$$

$$Z = C \div \frac{X}{A} \times \frac{Y}{B} = C \times \frac{A}{X} \times \frac{Y}{B} = \frac{CAY}{XB}$$

A9.5

Suppose 15 fishermen can catch 30 fish in 5 hours.

Fishermen	Fish	Time
15	30	5 hours
45	?	10 hours
?	60	20 hours
45	90	?

1. How many fish can 45 fishermen catch in 10 hours? Number of fish is directly proportional to the number of fishermen and directly proportional to time.

Calculation: $30 \times \frac{45}{15} \times \frac{10}{5} =$ ______________

2. How many fishermen are required to catch 60 fish in 20 hours? Number of fishermen is directly proportional to the number of fish, but inversely proportional to time.

Calculation: $15 \times \frac{60}{30} \div \frac{20}{5} =$ ______________

3. How long will it take 45 fishermen to catch 90 fish? Time is inversely proportional to the number of fishermen, but directly proportional to the number of fish.

Calculation: $5 \div \frac{45}{15} \times \frac{90}{30} =$ ______________

4. If a man-and-a-half can catch a fish-and-a-half in a day-and-a-half, how many fish can six men catch in seven days?

Fishermen	Fish	Time
1.5	1.5	1.5
6	?	7

Name____________________________Quiz1.1332

Give five examples of each of the following or explain why you can't:

1. Integers that are not positive.	2. Rational numbers that are not integers.
3. Real numbers that are not rational.	4. Integers that are not real numbers.
5. Terminating decimals.	6. Repeating decimals.

If 3 soldiers can dig 12 foxholes in 5 hours, how many soldiers are needed to dig 60 foxholes in 25 hours? How many foxholes can 9 soldiers dig in 15 hours? How long will it take for 6 soldiers to dig 24 foxholes? How many soldiers are needed to dig 18 foxholes in 150 minutes?

Soldiers	Foxholes	Time
3	12	5 hours
?	60	25 hours
9	?	15 hours
6	24	?
?	18	150 minutes

A 11

Name______________________________ Quiz 10.1332

1. For each of the following numbers, check all that apply.

	Natural Number	Integer	Rational Number	Irrational Number	Real number
$-\frac{123456}{2}$					
$.\overline{37}$					
$\sqrt{-144}$					
$\sqrt[3]{-144}$					
$\frac{1000000003}{2}$					
$\frac{2}{2\pi}-\frac{1}{\pi}$					
$-\frac{\pi}{9\pi}$					
$2^{123456789}$					

2. If 10 hairdressers can give 10 permanents in 10 hours, how many permanents can 15 hairdressers give in 15 hours? How many hairdressers are required to give 30 permanents in 5 hours? How long will it take 30 hairdressers to give 45 permanents?

Hairdressers	Permanents	Hours
10	10	10
15	?	15
?	30	5
30	45	?

A12

Name____________________________ Quiz 12.1332

1. For each of the following numbers, check all that apply.

	Natural Number	Integer	Rational Number	Irrational Number	Real number
$-\frac{123321}{3}$					
$.\overline{7324}$					
$\sqrt{-64}$					
$\sqrt[3]{-64}$					
$\frac{1000000002}{4}$					
$\frac{7\pi^2}{2\pi}$					
$6-\sqrt{36}$					
$10^{123456789}$					

2. If 15 pipe layers can lay 4 miles of pipe in 6 months, how many miles of pipe can 10 pipe layers lay in 2 years? How many pipe layers are required to lay 6 miles of pipe in 9 months? How long will it take 30 pipe layers to lay 48 miles of pipe?

Pipe layers	Miles of pipe	Time
15	4	6 months
10	?	2 years
?	6	9 months
30	48	?

A13

Name______________________________ Quiz 13.1332

1. For each of the following numbers, check all that apply.

	Natural Number	Integer	Rational Number	Irrational Number	Real number	Imaginary
$-\frac{123321456}{3}$						
$.\overline{3364}$						
$\sqrt{-49}$						
$\sqrt[3]{-6}$						
$\frac{34000000006}{4}$						
$\frac{9\pi^2}{5\pi}$						
$\frac{-6\pi}{2\pi}$						
$8-\sqrt{64}$						
$10^{1234567899871}$						

2. If 15 pipe layers can lay 4 miles of pipe in 6 months, how many miles of pipe can 30 pipe layers lay in 2 years? How many pipe layers are required to lay 32 miles of pipe in 9 months? How long will it take 10 pipe layers to lay 48 miles of pipe?

Pipe layers	Miles of pipe	Time
15	4	6 months
30	?	2 years
?	32	9 months
10	~~64~~ 48	?

A14

Name____

A student's GPA is calculated as follows.

(A) For each course, multiply the number of credit hours by the number of points associated with each grade, viz.,

A	B	C	D	F
4	3	2	1	0

Illustration. Suppose you get an "A" in Math 1332. Since this is a 3-hour course, and an A is worth 4 points for each credit hour, an A in Math 1332 will result in 4 x 3 = 12 points.

(B) Add the points earned by all of the classes.

(C) Divide by the total number of semester hours attempted, including F's but not IP's.

Example.

Course	Credit hours attempted	Grade/Multiplier	Points earned
Math 1332	3	$A \rightarrow 4$	12
Engl 1301	3	$B \rightarrow 3$	9
Hist 1301	3	$C \rightarrow 2$	6
Biol 1401	4	$D \rightarrow 1$	4
Totals	13		31
GPA = 31/13 = 2.3846153 ≈ 2.38			

Your turn. ☺

1. C. Cruz			
Course	Credit hours attempted	Grade/Multiplier	Points earned
Arts 1316		B	
Psyc 2306		C	
Govt 2301		B	
Chem 1305		A	
Chem 1105		F	
Totals			
GPA = = ≈			

2. B. Stille			
Course	Credit hours attempted	Grade/Multiplier	Points earned
Arts 1316		B	
Psyc 2306		C	
Govt 2301		B	
Chem 1305		F	
Chem 1105		A	
Totals			
GPA =		=	≈

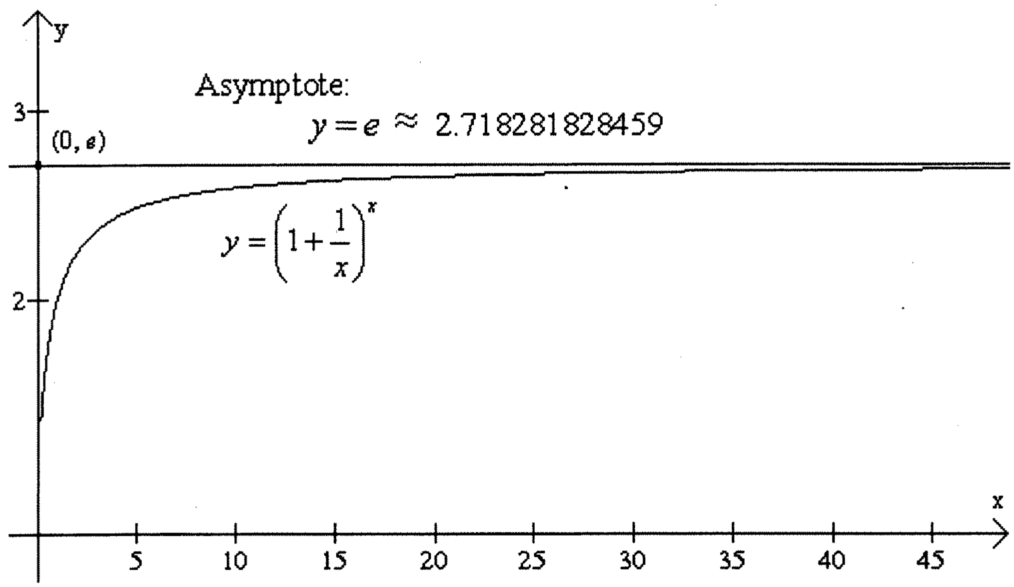
3. G. Whizz			
Course	Credit hours attempted	Grade/Multiplier	Points earned
Arts 1316		A	
Psyc 2306		A	
Govt 2301		A	
Chem 1305		A	
Chem 1105		A	
Totals			
GPA =		=	≈

Can you predict E. Holay's GPA before you calculate it?

4. E. Holay's			
Course	Credit hours attempted	Grade/Multiplier	Points earned
Arts 1316		A	
Psyc 2306		C	
Govt 2301		A	
Chem 1305		C	
Chem 1105		B	
Totals			
GPA =		=	≈

5. O. Well graduated with a GPA of 2.98, which she earned taking a total of 120 semester hours of work. Included is a statistics course, math 1442, in which she made a B. What would her GPA have been if she had made an A instead?

Consider the expression $\left(1+\frac{1}{x}\right)^x$ for positive values of x. As x gets larger and larger, the value of $\frac{1}{x}$ gets closer and closer to zero, so that $1+\frac{1}{x}$ gets closer and closer to 1. Of course the *exact* number 1 raised to any power is exactly 1, but if x is a positive real number and a is a real number *even a little bit greater than* 1, then a^x will also be greater than 1. If we look at the graph of $y=\left(1+\frac{1}{x}\right)^x$ we will note that it has a horizontal asymptote. Specifically, as x gets arbitrarily large, $\left(1+\frac{1}{x}\right)^x$ gets arbitrarily close to a number we call "e," an irrational number (like π), whose value is approximately 2.718281828459. This is illustrated by the figure below.



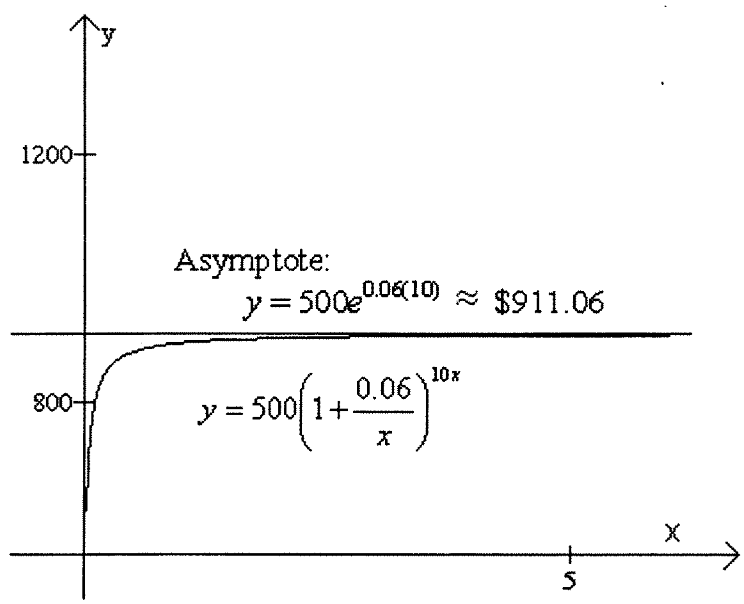
****

Let's see how this unusual number impacts everyday life. Recall the formula for compound interest: P dollars, at annual interest rate r, compounded n times per year for t years will grow to $P\left(1+\frac{r}{n}\right)^{nt}$ dollars. For example, \$500 at 6% compounded monthly for 10 years will grow to $500\left(1+\frac{0.06}{12}\right)^{12(10)} \approx \$909.70$.

The more often the interest is compounded, the larger the balance will grow. One might wonder if there is a limit to how large the balance can get. In particular, one might think that if one invested, for example, \$500—and if the interest were compounded often enough, perhaps a trillion times every nanosecond—the balance would grow to some enormous sum. Sadly, this turns out not to be the case. ☹

For example, the expression used to calculate the amount to which an investment of \$500 will grow, in 10 years, at 6% interest, if the interest is compounded x times per year is this: $500\left(1+\frac{0.06}{x}\right)^{10x}$

From the graph below, we see that the function $y = 500\left(1+\frac{0.06}{x}\right)^{10x}$ has a horizontal asymptote whose equation involves the number e. As x gets larger and larger, i.e., the interest is compounded more and more often, the value of $500\left(1+\frac{0.06}{x}\right)^{10x}$ will get arbitrarily close to $500e^{0.06(10)} \approx \$911.06$. This is an example of what is commonly called compounding the interest "continuously."



Notice that, with an initial investment of \$500, compounding the interest continuously for 10 years results in earnings of only \$1.36 more than compounding the interest monthly. This is reflected in the above graph, which shows the curve fairly close to its asymptote even when the interest is compounded only a few times a year.

P dollars invested at an annual rate r compounded n times per year for t years will grow to an amount $A = P\left(1+\frac{r}{n}\right)^{nt}$

Illustrations:

P	r	n	t years	A	Result of calculation for A
1000	6%	1	5	$1000\,(1+0.06)^5$	
1000	6%	2	5	$1000\left(1+\frac{0.06}{2}\right)^{2(5)}$	
1000	6%	6	5	$1000\left(1+\frac{0.06}{6}\right)^{6(5)}$	
1000	6%	12	5	$1000\left(1+\frac{0.06}{12}\right)^{12(5)}$	
1000	6%	365	5	$1000\left(1+\frac{0.06}{365}\right)^{365(5)}$	
1000	6%	1000	5	$1000\left(1+\frac{0.06}{1000}\right)^{1000(5)}$	

Notice that the more times per year the money is compounded, the larger the value of A. But there is a limit to how large the money can grow, even if the money is compounded *continuously*, meaning an infinite number of times, in which case it will grow to $A = Pe^{rt}$

For example, \$1000 at 6% compounded continuously for 5 years will grow to: $A = 1000e^{0.06(5)} =$ ____________.

A19

If the investment is compounded continuously, determine the value of A in each of the following cases.

P	r	t years	A	Result of calculation for A
1000	3.5%	10	$1000e^{0.035(10)}$	
1000	4%	10	$1000e^{0.04(10)}$	
1000	6%	10	$1000e^{0.06(10)}$	
1000	8%	10	$1000e^{0.08(10)}$	
1000	10%	5		

1. Determine how long it would take for $1000 to grow to $2000 at 6% if it is compounded continuously. Round answer to the nearest year.

2. Determine how long it would take for $1000 to grow to $2000 at 8% if it is compounded continuously. Round answer to the nearest year.

3. Determine how long it would take for $1000 to grow to $2000 at 9% if it is compounded continuously. Round answer to the nearest year.

4. In each of the 3 previous problems, multiply your answer by the corresponding interest rate. What do you notice?

5. What simple interest rate is equivalent to 6% interest compounded continuously? Hint: determine the amount that $1 would grow to if compounded continuously.

Name________________________ A 20

On a separate sheet, write the appropriate calculation, then put the result in the space below.

1	What is the APY* for money invested at 8.1% compounded continuously?
2	Based on a 365-day year, what is the APY for money invested at 8.1% compounded daily?
3	What annual nominal rate compounded continuously has the same annual percentage yield as 7.1% compounded monthly?
4 (a) (b)	If $1.00 had been placed in a bank account in 1945, how much would be in the account at the end of 2010 if the money earned (a) 3.1% compounded annually or (b) 3.1% simple interest**?
5	A zero coupon bond with a face value of $1000 matures in 10 years. What should be bond be sold for now if its rate of return is to be 3.194% compounded annually?
6	How long will it take $12,000 to grow to $16,000 if it is invested at 8.1% compounded quarterly? (Round up to the next-higher quarter if not exact.)
7	In how many months will money invested at 7.1% compounded monthly grow to an amount greater than what it would at 10% simple interest?
8	If $20,000 is invested at 4.1% compounded monthly, what is the amount after 5 years?
9	How long will it take money to quadruple if it is invested at 6.1% compounded continuously?
10	Using compound interest formula $$A = P(1+i)^n$$ determine P if A = $60,000, i = 0.0051 and n = 70.

*Annual Percentage Yield

**Not compounded at all, so that $A = P + PRT$