Determining the day of the week for virtually any date of interest, past or future

STEPS	Example: Jan 23, 1971
If and only if the given month is **January or February**, use the year PRIOR to the actual year given. Thus, in this example, **MONTH = January, DAY = 23, and YEAR = 1970** (not 1971).	

STEPS	Example: Jan 23, 1971
Begin with the number associated with MONTH in the following chart.	**January: 4**

Jan: 4	Feb: 0	Mar: 6
Apr: 2	May: 4	Jun: 0
Jul: 2	Aug: 5	Sep: 1
Oct: 3	Nov: 6	Dec: 1

STEPS	Example
Add DAY to the number you got in the previous step.	**4 + 23 = 27**

If YEAR is in the	then add:
1700's	2
1800's	0
1900's	5
2000's	4

(N.B. The sequence 2, 0, 5, 4 repeats, so the number for the 1600's is 4)

1970 is in the 1900's so add 5.

27 + 5 = 32

STEPS	Example
Add the number formed by the last two digits of YEAR.	**32 + 70 = 102**
Add the quotient of the last two digits of YEAR divided by four (ignore the remainder)	**70 ÷ 4 = 17.5 (use 17)** **102 + 17 = 119**
Divide the result by 7 and NOTE THE REMAINDER.	**119 ÷ 7 = 17 (remainder 0)**

If the remainder is	the day fell on:
1	Sunday
2	Monday
3	Tuesday
4	Wednesday
5	Thursday
6	Friday
0	Saturday

Since the remainder is 0,

January 23, 1971 was a Saturday

Extra credit: Show that the first day of any new century, such as January 1, 2001 (or 2101 or 2201 etc.) must fall on only one of four days of the week.

Here's a little calculator exercise to get your fingers warmed up.

If you're using a calculator that "knows" order of operations (such as a scientific calculator) be sure to hit "=" at the end of each of the 14 steps below. If your calculator doesn't know order of operations, hitting the "=" is optional. Here's how you can tell.

Turn your calculator on and punch in: $3 + 5 \times 7 =$

- If your calculator says 56, it **doesn't** know order of operations. Instead it does the operations in the same order you enter them, so hitting "=" at the end of each step below is optional.

- If your calculator says 38, it **does** know order of operations, so hitting "=" is mandatory at the end of each of the 14 steps below.

- If your calculator says anything else, see your instructor. ☺

This exercise involves the first six primes: 2, 3, 5, 7, 11, and 13.

Write down any 7-digit number, such as your phone number. Now, let **x represent the number formed by the first three digits** and **y represent the number formed by the other four digits**. For example, if your phone number is **567-1892, then x = 567 and y = 1892**. (If your number is 567-0002, then y would just be 2.)

1. Enter x into your calculator.

2. Multiply by 2, then add 2.

3. Multiply by 3, then add 3.

4. Multiply by 5, then add 5.

5. Multiply by 7, then add 7.

6. Multiply by 11, then add 11.

7. Multiply by 13, then add 13.

8. Subtract 27.

9. Divide by 30.

10. Subtract x.

11. Subtract 1,111.

12. Multiply by 10.

13. Add y.

14. Subtract 5,950.

You should now be looking at your original 7-digit number. If not, try it again! For a real challenge, try figuring out how it works!

A single move by the chess piece known as the Knight consists of one space horizontally and two spaces vertically or two spaces horizontally and one space vertically. Show that it is possible for a Knight to traverse a 10 by 10 grid by landing on each of the 100 locations exactly once. Initially, you may place your Knight on any square you choose. Define the path by entering the integers from 1 to 100 in the table below.

For extra credit, (1) describe how this puzzle might have been created in the first place and (2) state how large a grid the Knight might be able to traverse in this way.

If you want to work on this without a hint, don't read the hint below or look at the next page!

Hint: Try solving the problem for a five-by-five grid first.

The problem can be solved in more than one way. One solution is to first traverse a 5 by 5 grid, then place it in the lower left corner of the 10 by 10 grid (see below). At this point, the problem is essentially solved because, using the 5 by 5 as a guide (with suitable rotations or reflections), one could start and end the other three 5 by 5 grids as indicated below.

100					75				
				76					51
7	16	21	12	5			50		
22	11	6	15	20					
17	8	13	4	25					
10	23	2	19	14					
1	18	9	24	3	26				

The negation of a statement is another statement with the opposite truth value of the original. Illustrations follow.

Statement	Negation
John is cool.	John is not cool.
Everyone is correct.	Not everyone is correct. Or: at least one person is not correct.
Sue likes liver.	Sue does not like liver.
John or Sue is over 21.	Neither John nor Sue is over 21.
No one is going to pass.	Someone is going to pass.
Some people are not weird.	Everyone is weird.
No one doesn't like Sara Lee	Someone doesn't like Sara Lee
John and Sue are both old.	Either John or Sue is not old. Note: "or" means either one or both.
Everything is possible.	Something is not possible.

Okay, write the negation of each of these statements below.

Statement	Negation
B is not cool.	
Either B or C is not cool.	
At most two of the three are cool.	

If one of the statements is true, one is false, and the third is unknown, what does that imply about the three negations, and how does that help?

Five men have the first names Alex, Darcy, Evelyn, Francis, and Hilary. Their surnames are also Alex, Darcy, Evelyn, Francis and Hilary. Their wives are also Alex, Darcy, Evelyn, Francis, and Hilary, but nobody has the same first name as surname. Hilary is married to Hilary, and their surname is the same as the first name of Mr. Hilary. Mr. and Mrs. Hilary share the same first name. Alex married Darcy (their surname is not Mr. Darcy's first name), but the other Alex did not marry Evelyn.

What are Francis's wife's first name and surname?

Given names							Surnames				
WOMEN→ MEN↓	A	D	E	F	H		A	D	E	F	H
A											
D											
E											
F											
H											

Given names							Surnames				
WOMEN→ MEN↓	A	D	E	F	H		A	D	E	F	H
A											
D											
E											
F											
H											

Given names							Surnames				
WOMEN→ MEN↓	A	D	E	F	H		A	D	E	F	H
A											
D											
E											
F											
H											

For a complete explanation of how to use this table to solve the problem, go to the next page.

Given names							Surnames				
WOMEN→ MEN↓	A	D	E	F	H		A	D	E	F	H
A											
D											
E											
F											
H											

Illustration: since no one has the same given name as surname, we can eliminate names like "Alex Alex" from consideration. Since Hilary is married to Hilary, neither can be married to anyone else.

Given names							Surnames				
WOMEN→ MEN↓	A	D	E	F	H		A	D	E	F	H
A					No		No				
D					No			No			
E					No				No		
F					No					No	
H	No	No	No	No	Yes						No

Here is one possible solution. Letters indicate things that are ruled out. Numerals indicate valid conclusions.

To illustrate how to read the table, note the "2" and the "10" across from the "A" on the left and under the "D" and "F." This means Alex the man is married to Darcy and their surname is Francis.

For details, refer to the notes corresponding to the letters and numerals below the table.

Given names						Surnames				
WOMEN→ MEN↓	A	D	E	F	H	A	D	E	F	H
A	X	2	X	X	Y	Z	X&Z	M	10	V
D	S&Z	X	N	7	Y	3	Z	T	T	V
E	W	X	6	P	Y	S	P	Z	T	5
F	4	X	R	R	Y	S	9	T	Z	Q
H	Y	Y	Y	Y	1	U	U	8	M	Z

Z: No one has the same first name as surname.

1: Hilary is married to Hillary.

Y: Neither Hillary is married to anyone else.

2: The statement "Alex married Darcy" is ambiguous. Let's try to work the problem assuming that the statement refers to Alex the man and Darcy the woman.

X: Alex and Darcy aren't married to anyone else. This marriage also eliminates the possibility of Alex's last name being Darcy.

X&Z: Because Alex married Darcy, their last name can't be Darcy; otherwise, she would be Darcy Darcy.

W: The other Alex did not marry Evelyn.

V: Because Mr. and Mrs. Hillary share the same first name, and because Alex married Darcy, the surnames of Alex and Darcy cannot be Hillary.

U: By process of elimination, Mr. Hillary's given name must be Evelyn or Francis. Therefore Hillary's surname must be Evelyn or Francis, not Alex or Darcy.

T: This is tricky. By process of elimination, the surnames of the men Alex and Hillary must be Evelyn and Francis. Therefore it must be either "Alex Evelyn and Hillary Francis" or vice versa. *Thus, none of the other three men can have Evelyn or Francis for their surname.*

3: By process of elimination, the last name of the man Darcy must be Alex.

S: The only man whose last name is Alex is Darcy.

S&Z: Mr. Darcy Alex cannot be married to Alex because she would then be Alex Alex.

4: By process of elimination the man Francis is married to Alex. (Note: we now believe that Francis's wife is Alex, which is part of the question we're trying to resolve; it remains to discover her last name.)

R: Being married to Alex, the man Francis cannot be married to Evelyn or Francis.

Q: Since Mr. & Mrs. Hillary share a given name, it can't be Francis because he is married to Alex.

5: By process of elimination, Mr. Hillary's first name must be Evelyn.

6: Since the Hillary's share first names, her name must be Evelyn also.

P: Being Evelyn and Evelyn Hillary, he can't be married to Francis and his last name can't be Darcy.

7: By process of elimination, the woman Francis is married to Darcy.

N: Since he is married to Francis, Darcy isn't married to Evelyn.

8: Because it's Evelyn and Evelyn Hillary, and because Hillary and Hillary's surname is the given name of the Hillary's, it must be Hillary and Hillary Evelyn.

M: Hillary's surname being Evelyn means Alex and Darcy can't have Evelyn for their surname, nor can Hillary have Francis for a surname.

9 & 10: Process of elimination.

Hence, to answer the question, Francis's wife is Alex Darcy. Or is it? Let's check all the criteria.

All is well until we get to this:

Alex married Darcy (their surname is not Mr. Darcy's first name),

Notice that Alex and Darcy's surname is Francis, and Mr. Darcy's first name is also Francis!

So what happened? Recall that we assumed that the man Alex married the woman Darcy.

Now go back and assume that the woman Alex married the man Darcy and see if you get a solution. ☺

Given names						Surnames				
WOMEN→ MEN↓	A	D	E	F	H	A	D	E	F	H
A										
D										
E										
F										
H										

1. Four people, Bono, Edge, Adam, and Larry, are trying to cross a bridge at night with only one flashlight.

Bono can cross the bridge in 1 minute, Edge in 2 minutes, Adam in 5 minutes, and Larry in 10 minutes. Here are the rules.

- Two people cross the bridge
- One person returns with the flashlight
- Two more people cross
- One person returns with the flashlight
- Two more people cross, at which time all 4 of them are on the other side
- They always cross the bridge at the rate of the slower person.
- They have only 17 minutes!

So how do they do it?

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2. This Week: Two Bedouins, a Stranger and a Wandering Magistrate

RAY: Two Bedouins were traveling across the desert to a distant village. In the middle of the day, they sat down to eat the loaves of bread that they had brought with them for lunch. One of them had five loaves and the other had three.

Just as they were ready to eat, a stranger comes along and asks if he might share their meal.

He said he had plenty of money but no food. The two agreed to divide their loaves equally among the three of them.

After the meal was finished, the stranger laid down eight coins of equal value for what he had eaten and he went away. The traveler who had five loaves took up five coins and left three for the other guy. But the other guy disputed it, saying, "We shared the bread, we should each get four coins." Since they could not agree, they called in a magistrate.

TOM: Let me get this straight. They're out in the middle of the desert and there happens to be a wandering magistrate?

RAY: It happens. The magistrate listened to the story and then figured out who should get what. The question is, who's right? Or, is neither of them right?

46A

3.
There are 900 students and 900 lockers. At first, all the lockers are closed. As the first student enters the building, he opens each locker. The second student closes every even numbered locker. The third student changes every third locker (3, 6, 9, etc); that is, she opens the closed lockers and closes the opened ones. The fourth student does the same to every fourth locker (4, 8, 12, etc). Each student, in turn, does the same. That is, the nth student changes the condition of each nth locker, beginning with locker n. After the 900th student is done, which lockers are open?

4.
RAY: You have in your possession two pieces of string, each of which is a couple of feet long. The strings can both be different lengths. It doesn't matter. They're burnable, like a fuse that's used to light dynamite.

You could light either end of either string, and it would burn. In fact, if you lit one end of a string, it would burn in exactly an hour.

TOM: I'm with you. And I like it. Keep going, man!

RAY: Okay. You light one of the strings with your Zippo lighter.

TOM: It would burn right along and it would take one hour to be gone.

RAY: Right. But here's the wrinkle: the strings do not burn at a constant rate. For example, the string might burn for two minutes and then go crazy and burn like mad, then slow down, et cetera. In other words, you don't know at what rate the string's burning, at any specific time. All you know is that in an hour's time, the whole string is burned. It's not linear, nor is it predictable.

The question is, with the Zippo lighter and these two strings, how would you measure 15 minutes of time?

By the way, I think that there are two answers to this puzzler.

46 B

5. This Week's Puzzler: From the Department of Very Curious Math

RAY: After the long, miserable spring we had here in the great Northeast, we finally had a nice sunny, warm day. It happened to coincide with my 12-year-old son's being home from school on summer vacation. I asked him if he wanted to go for a ride, and he said, "Not really, Dad."

Then I said those magic words: ice cream. He quickly changed his mind.

As were riding over to the nearby creamery, he turned to me, and said, "I know that one and six is six. But, is two and five 10?"

"No," I replied, "two and five is 12."

"Then what's three and three?" he asked.

"17," I responded.

"Oh yeah," he said. "I think I've got it now."

The question is, what the heck were Tony and his son talking about?

TOM: Let me see if I've got this right. They're in the car. They're driving to the creamery. Is that right?

RAY: I didn't precisely say that. And that's a hint.

46 C

6.

RAY: I'm going to hand you a deck of normal playing cards—52 cards and no jokers. You're going to be placed in a darkened room. You'll also be blindfolded, and you'll be naked.

TOM: Are you going to stick pokers in my eyes, too?

RAY: The deck of cards will have 13 of the 52 cards turned face up. They're all mixed up in the deck. You don't know where they are. And you can't tell.

TOM: Because I'm blindfolded.

RAY: Right. Your challenge is to arrange the cards in two piles so that each pile has the same number of cards facing up.

TOM: How can I do that? There are 13 of them.

RAY: That's your problem. But here's a hint: They don't have to be equal piles. But, if the main deck winds up with, say, 9 face-up cards, the other pile has to have 9 face-up cards, too.

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RAY: Tommy, Dougie and I are sitting around the office one day at Car Talk Plaza. We were noticing how dingy the place looked. We'd been there 15 years, and the place had never been painted. So, we decided to paint Car Talk Plaza.

We didn't know which team of us was going do it, so we sat down and decided to do a little math. We determined that Tommy and I together could paint the entire Car Talk plaza in 10 days. After all, we had a lot of painting experience as kids, having painted Dad's car a couple of times with brushes.

Dougie and I could do it in 15 days. And, if Doug and Tom worked together, they could do it in 30 days.

The question is how long would it take each of us, painting by ourselves, to paint the whole of Car Talk Plaza?

46D

1. (Algebra problem most students find very difficult.) From point A, a woman rows upstream for 4.5 miles. There, she passes a hat floating downstream. She continues upstream for 1.5 hours, turns around, rows downstream, and catches the hat at point A. What is the speed of the current? (In addition to the rather difficult algebraic solution, there is also a rather creative solution requiring very little calculation--i.e., most people can do it mentally!)

2. (The "obvious" answer is wrong.) A bottle of wine costs $10. The wine itself (without the bottle) costs $9 more than the empty bottle. What is the cost of the empty bottle?

3. (See previous problem.) Jenny and Johnny together have $1.10. Jenny has one dollar more than Johnny. How much does Johnny have?

4. (Students find this hard.) If $1\frac{1}{2}$ hens lay $1\frac{1}{2}$ eggs in $1\frac{1}{2}$ days, how many eggs will three hens lay in three days?

5. (The "obvious" answer is wrong.") Suppose Sue and Joe race for 100 meters and Sue wins by five meters. They decide to race again, but this time Sue backs up five meters behind the starting line. If they each run at the same average speed as they did the first race, who will win the second race?

6. (Are there two answers?) How many pets of each kind does Tony have if all are dogs except two, all are cats except two, and all are parrots except two?

7. If a doctor gave Maria three pills and told her to take one every half hour, how long should it be between the taking of the first pill and last pill?

8. (Can you solve the first part of this without using the "beat-it-to-death" method? Twelve words are sufficient!) How many matches must be scheduled in a single-elimination tennis tournament with 31 entrants? What if it's double-elimination?

9. (Easy pigeon-hole-principle problem.) Show that in a room of 400 people, at least two have a birthday on the same month and day.

10. Show that in Bexar County at least two nonbald people have exactly the same number of hairs on their heads. (Hint: on the average, redheads have 90,000 hairs, brunettes have 120,000 hairs, and blondes have 130,000 hairs.)

11. (The "obvious" answer is correct.) If you drive two hours at 60 mph and two hours at 40 mph, what is your average speed for the four hours?

12. (The "obvious" answer is wrong.) If you drive to Houston at 60 mph and return at 40 mph, what is your average speed for the entire trip?

13. (It's less than 40 minutes.) What is the minimum amount of time to cook three steaks ten minutes on each side if the grill holds only two steaks at a time?

14. (Isomorphic to problem 13.) John and Betty have three tasks to perform: feed the baby, cut the grass, and vacuum the house. Each task takes 30 minutes. Only one person can work on a particular task at a time. What is the least amount of time to finish all three?

47

15. (Trick question.) How can two mothers and two daughters divide 21 pennies so that each receives the same number of pennies?

16. (Some people will never believe the answer, no matter how hard you try.) Imagine you have a cup of coffee and a cup of milk, with equal amounts of liquid in the two cups. A spoonful of milk is transferred from the milk cup to the coffee cup. The coffee is stirred and so is homogeneous. A spoonful of the mixture is returned to the milk cup, so that at the end, the amount of liquid in the two cups is still the same. Is there more milk in the coffee cup or more coffee in the milk cup or what? Guess what? The answer is the same, even if the mixture is not stirred!!!

17. (This is probably a hard problem.) Some time ago, the NFL consisted of two conferences of 13 teams each. The rules of the league specified that during the 14-week season, each team would play 11 games against teams in its own conference and 3 games against teams in the opposite conference. Prove that this is impossible--which explains why the NFL never satisfied its own scheduling rule!

18. (Clever, rather difficult.) A woman usually takes the 5:30 train, arriving at her station a half hour later, when her husband picks her up and drives her home. One day she took the 5:00 train which got her to the station a half hour earlier than usual. She began walking home. Her husband, starting out from home to meet her at the usual time, met her on the way and they arrived home 10 minutes before they usually did. How long did the woman walk? Assume that there is no ambiguity in the information so that an answer is possible.

19. (Easy for some.) You are in charge of maintaining a pond in which water lilies grow. Each day the area of the plants doubles. You decide to do nothing until the pond is half covered. Only then will you cut the water lilies back. Given that the entire pond is just covered on the one hundredth day, what day do you start to cut?

20. (Your suspicions are confirmed!) Tim has a contract which pays $5,000 every six months plus a $100 raise every six months beginning with the second six months. Carol has a contract which pays $10,000 per year plus a $400 raise every year after the first. Who gets more money?

21. A rectangle is inscribed in a circle. The midpoints of the sides are connected to form a rhombus (a parallelogram with equal sides). The distance from the center of the circle to one of the nearer vertices of the rhombus is five meters. The distance from this vertex to the circle is four meters. How long is each side of the rhombus?

22. How can you cut a cake into eight equal pieces by making only three straight cuts? (The cake is a right circular cylinder.)

23. (Think globally.) Beginning at the North Pole, if you fly 100 miles south, 100 miles east, and then 100 miles north you will be back at your starting point. From what other places (plural!) on earth is this possible?

24. Can you determine a person's seven-digit phone number by asking only 24 questions, each of which will be answered truthfully by "yes" or "no"?

25. This group of words is most unusual. Why? If you look at it, you can probably find out, but a solution is not all that obvious.

26. Bob's sister, Debbie, has one more sister than she has brothers. How many more sisters than brothers

does Bob have? (Bob is Debbie's brother, not her sister.)

27. How can you seal 1000 pennies in ten plastic bags so that you can give someone any number of pennies from one to 1000 (exactly) without opening any bag?

28. Suppose you have ten glasses in a row. The first five are full of water; the rest are empty. Can you move only two glasses so that the full and empty glasses alternate?

29. (Diophantus would be proud if you can solve this one without guessing!) If cows are $10, sheep are $5, and pigs are $0.50, can you buy exactly 100 of these animals by spending exactly $100?

30. This sentence has three erors.

31. Can you place 44 pennies in 10 pockets so that each pocket contains a different (whole) number of pennies?

32. If it takes five seconds to strike six o'clock, how long will it take to strike twelve o'clock?

33. A taxi driver takes a passenger from the Waldorf to Kennedy Airport at 30 mph in 80 minutes. He takes a passenger back over the same route at the same speed in one hour twenty minutes. Why?

34. (Some people hate this one.) Why are 1981 dollar bills more valuable than 1976 dollar bills?

35. How can an auto race be held in which the owner of the car finishing last wins?

36. *N.Y. Times*, May 6, 1965: "Albany kills bill to repeal anti-abortion law." Did this make abortion legal or not?

37. (This is a rotten problem.) Rearrange the letters in "new door" to make one word.

38. (So is this.) Sign on post unearthed in Roman ruins: "Toti emul esto." What was post used for?

39. Suppose there are three boxes, each with exactly two coins inside. One contains two nickels, one two dimes, and one a nickel and a dime. The boxes are labeled "10 cents," "15 cents," and "20 cents," but all the labels are wrong. Someone takes a coin out of the box labeled "15 cents" and places it in front of you. Knowing only this coin, determine the contents of each box.

40. Pat Summerall, on protecting the quarterback: "If you take the violence out of football, the stands will be filled with no-shows." Precisely what does that mean?

41. (Not an easy problem.) Begin with a rectangle 7 by 6 inches. Cut out one-inch squares from the ends of one seven-inch side so that 40 square inches remain. Can this figure be tiled with 20 one-by-two-inch dominoes without breaking any dominoes?

42. A pawn shop sold 30 old glasses at two for a dollar and another 30 at three for a dollar. This brought in $25. The next day, the clerk put another 60 glasses on the counter and said, "Why bother to sort them? If 30 sell at two for a dollar and 30 at three for a dollar, I'll just sell them all at five for two dollars." This brought in only $24. Why?

43. During the first half of the baseball season, Charlie Slugger batted 53 for 83 while Davy Popout was

64 for 101. The second half, Charlie was 67 for 158 while Davy was 38 for 90. The owner agreed to pay a bonus to the better hitter. Charlie said he should get it since his average was better in each half of the season. Davy said he should get it since Charlie was only 120 for 241 for the year, while he was 102 for 191. Who's right?

44. (Not for the faint of heart!) Five men and a monkey are marooned on a desert island. They spent the first day gathering coconuts. During the night, one man decided to take his share. He divided the coconuts into five equal shares, with one coconut left over which he gave to the monkey. He hid his share and put the rest back together. The other men in turn did the same. Each had one coconut left over which he gave to the monkey. In the morning, the remaining coconuts were divided into five equal shares, with none left over for the monkey. How many coconuts were there to begin with?

45. Tom the Truther, Larry the Liar, and Curly the Confused are sitting alongside each other in a movie. Tom always tells the truth, Larry never tells the truth, and Curly sometimes tells the truth. The man on the left says, "The guy in the middle is Tom." The man in the middle says, "I'm Larry." The man on the right says, "The man in the middle is Curly." What are the correct names of the three men?

45.5 Suppose one of the three people in the last problem knocks on your door. When you answer the door, he says, "Hi, I'm Larry." Who is he? Could problem 45 have been solved if the man on the right did not say anything?

46. (This problem can be solved with a 10-word sentence, which, of course, must be verified.) You are skiing. You come to a point where three slalom courses branch off: A, B, and C. To tell skiers whether the slopes are safe, there is a computer there with three data banks. But you know that saboteurs have rigged the computer so that one data bank always tells the truth, one always lies, and one alternates between truth and falsehood. The three data banks say:

#1. At most, two of the slopes are safe.

#2. Slope B is dangerous.

#3. Slope B or Slope C is dangerous.

Which slope, if any, is safe?

47. Try this on your algebra (and above) classes.

Suppose:

a. Exactly 90% of people surveyed bathe at least once a week.

b. Exactly 92% of the females in the survey bathe at least once a week.

c. Exactly 65% of the males in the survey bathe at least once a week.

What fraction of those surveyed are males who do NOT bathe at least once a week? (When one of these guys goes to a restaurant, the waiter misspeaks: "May I take your odor?")

50

48. (Geometry anyone?) There are three lighthouses, A, B, and C, whose positions are known and plotted on your chart. You are in a ship at unknown point S. Knowing $\angle ASB$ (call it α) and $\angle BSC$ (call it β), use only a compass and straight edge to plot your position, S, on the chart.
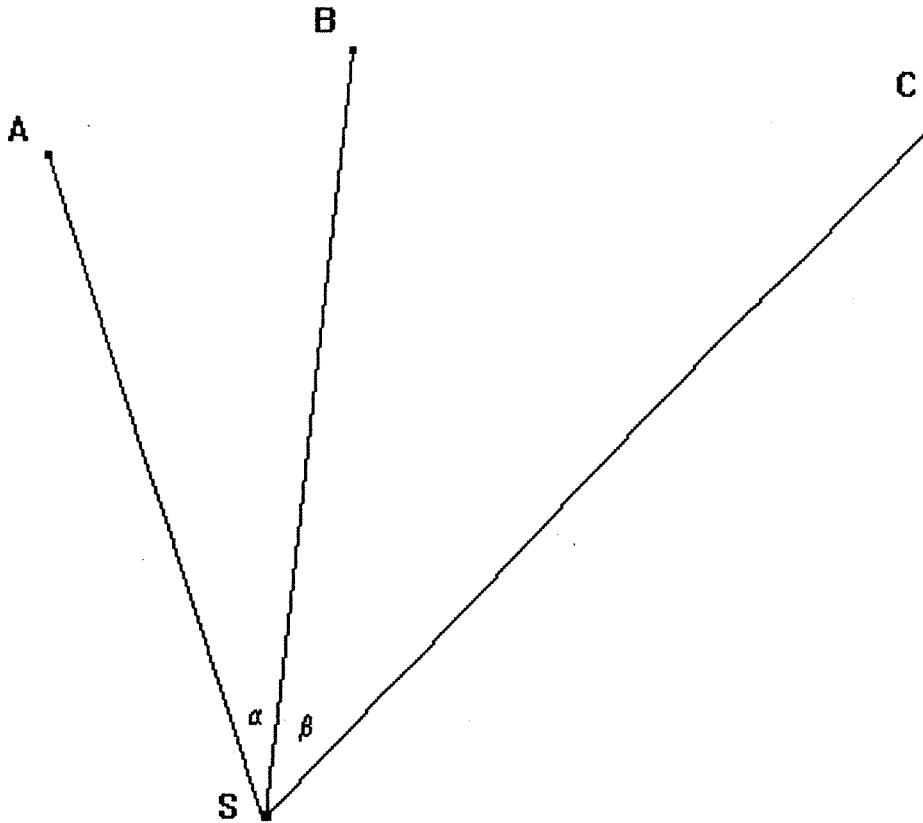
49. (Cf. Marilyn Savant.) Suppose you're on a game show and you select a prize by choosing one of three doors, A, B, or C. Behind two of the doors is a consolation prize; behind the other one is a nice prize. After you choose your door, the host will show you what's behind one of the other doors. (He will always show you a consolation prize.) Suppose you choose door A. The host then shows you that door B hides a consolation prize, and gives you the option of staying with door A or switching to door C. Is there any advantage to switching?

50. Same situation as last problem, except there are five doors. Obviously, you have a one-in-five chance of winning. You pick a door. The host then shows you, one at a time, three losing doors and gives you the option of staying with your original choice or switching to the last door. Of course, you have either picked the winning door or not (and the host knows which) so that there is no longer a probability involved from the host's point of view, but from *you're* point of view, what is your probability of winning if you don't switch? If you do switch?

51. (Good intro for direct and inverse variation, not to mention common sense.) If 2 soldiers can dig 3 foxholes in 4 hours, how many soldiers would it take to dig 600 foxholes in 20 hours?

52. (Many people don't believe the answer even after being shown a proof.) At noon on Monday, a salesperson leaves San Antonio for Austin. At noon on the following day, she leaves Austin and returns to San Antonio. She travels the same route (only in the opposite direction) as she traveled the previous day. The answer to both of the following questions is yes. Can you prove it?

    a.   Is it POSSIBLE for her to arrange it so that she is at some point on the highway (but in the other lane, of course) at the same time she was there the previous day (only 24 hours later)? (This is the easy part.)

    b.   Does there necessarily EXIST a such point on the highway? (This is the hard part.)

53.   Three schools have a track meet and enter one person in each of the events. The number of events is unknown, and so is the scoring system--except that the winner of each event scores a certain number of points, second place scores fewer points, and third place scores fewer still (but not zero). Georgia won with 22, and Alabama and Florida tied with 9 each. Florida won the high jump. Who won the mile run?

54.   Write the numbers 1, 2, 3, 4, 5, 6, 7, 8 (one in each rectangular section) so that no two rectangles that touch--even in one point--contain consecutive integers.
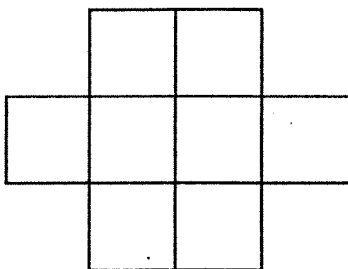
55.   Write the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 (one in each rectangular section) so that the total of every row, every column, and both diagonals is the same. (Hint: first figure out what the total must be.) For a real challenge, try solving the problem for, say, a 7 by 7 square using the numbers 1, 2,..., 49.
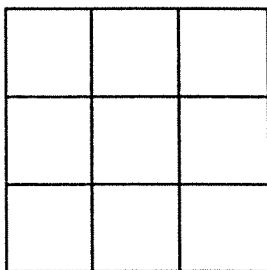
56.   The figure below represents a house with five rooms and 16 doors. Prove that it is not possible to walk through each door exactly once. Tricky solutions, such as going through or inside walls, are not allowed.
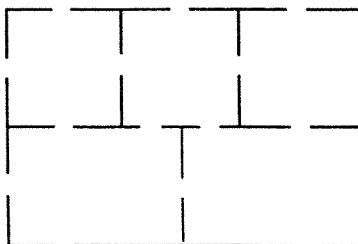
57. A farmer wishes to stake out a goat at point P on the fence enclosing a one-acre circular field so that the goat can eat one-half acre of grass. How long should the rope be? Answer: approximately 136.4428 feet. Prove it.

P
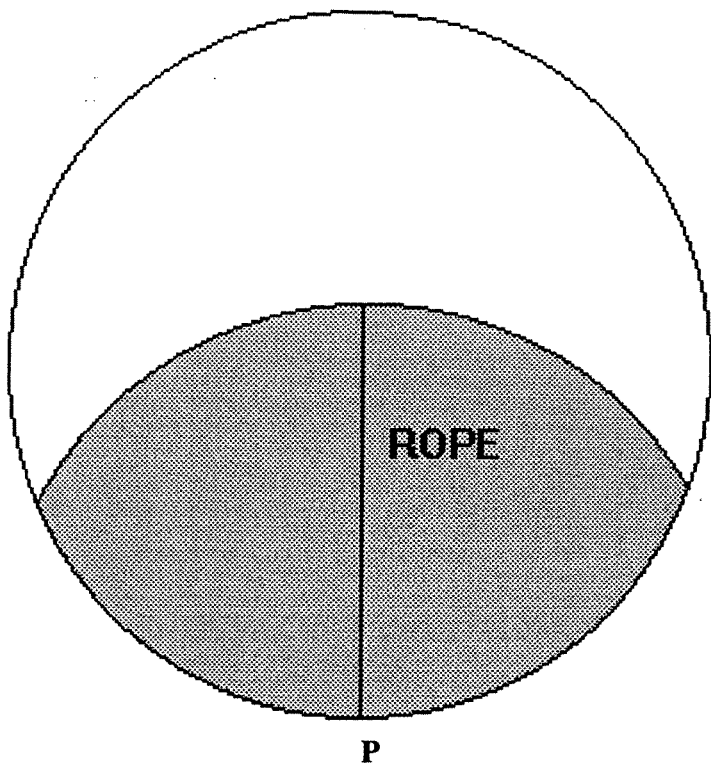
58. The sum of the ages of Smith and Wesson is 49. Find the age of each if the current age of Smith is twice that which Wesson was when Smith was as old as Wesson is now.

59. The sum of the ages of Curly and Larry is 92. Find the age of each if four years from now, Curly will be twice as old as Larry was when Curly was as old as Larry is now.

60. Suppose that exactly 90% of adults surveyed are in favor of health care for all, including exactly 92% of the women surveyed and exactly 65% of the men surveyed. What fraction of the adults surveyed are men who are not in favor of health care for all?

61. Suppose you are in Rome, Italy and you find a coin dated 26 bc. What would you do to find out if it is authentic?

62. Suppose you got a job that lasted 20 days. You could choose to be aid either $10,000 or in the following way: first day, one cent; second day, two cents; third day, four cents; fourth day, eight cents; fifth day, sixteen cents; and so on for twenty days. Which would be more money after twenty days?

63. How can you plant ten trees in five straight rows with four trees in each row?

64. Name the next three logical letters in the sequence: OTTFFSSEN.

65. Three men check into a hotel. The clerk charges them $30 for a room and they each give him $10.

The clerk later discovers that he gave them a $25 room, so he gives the bellhop $5 to return to them. Since the bellhop doesn't know how to divide the $5 equally among the three men, he gives each man one dollar and keeps the extra two dollars. Now each man has paid $9 for the room for a total of $27 and the $2 for the bellhop makes $29. What happened to the other dollar?

66. A man has to take a wolf, a goat, and some cabbage across a river. His rowboat has enough room for the man plus either the wolf or the goat or the cabbage. If he takes the cabbage with him first, the wolf will eat the goat; if he takes the wolf with him first, the goat will eat the cabbage. Only when the man is present are the goat and cabbage safe with their enemies. How can the man get the three across the river safely?

67. "How much does one cost?" "Fifty cents," replied the hardware store clerk. "And how much does 18 cost?" "One dollar." "Okay, I will take 706." "That will be $1.50." What was the customer buying?

68. Because he likes to fish, Mike just got a job that requires him to work 12-hour shifts, four consecutive days, followed by four days off. His friend, Bill, has a similar job with similar pay, but works the usual schedule of eight-hours on, sixteen off, five days a week. On the average, who works more hours?

Name_____

Suppose you wanted to make a two-dimensional scale model of the solar system and you represent the earth as a circle one inch (2.54 cm) in diameter.

1. About how large a circle should represent the moon? (Answer in inches and cm).

2. About how far away from the earth's circle should the moon be placed? (Answer in inches and cm).

3. About how large a circle should represent the sun? (Answer in feet and meters).

4. About how far away from the earth's circle should the sun be placed? (Answer in feet and meters).

5. If you decided to include the star nearest the earth, about how far away from the sun's circle should it be placed? (Answer in miles and km).

First, find the following information and write the data in miles in the left margin. As a class, we will decide what approximations to use and fill in the blanks together.

	Miles	Kilometers
diameter of earth		
diameter of moon		
diameter of sun		
distance from earth to moon		
distance from earth to sun		
distance from earth to nearest star (find answer in light years, then convert to miles and kilometers)		

Allstate.
You're in good hands.
8550 Ulmerton Rd, Ste 162
Largo, FL 33771-9933

July 20, 2005
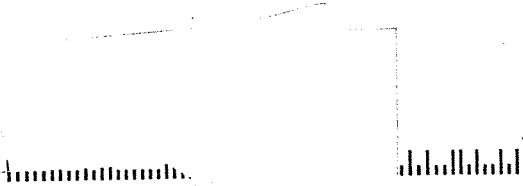
RE: Policy Number: 638576793

One of the many factors that affect how much you pay for your automobile insurance is the usage of your vehicles.

As you can imagine, accurate information helps us make sure that the vehicles you insure with us are rated properly. That's why we have enclosed an Estimated Annual Mileage and Usage Verification Form (referred to below as the "Form"). **Completing, signing, and returning this form to us within the next 20 days will help ensure that your auto policy is accurately rated.**

When completing the Form, please provide information for only those vehicles listed. **The Form may not list all of your insured vehicles.** We will consider the information you provide on the Form before we calculate the premium for your upcoming policy renewal offer if we receive the completed form from you within the next 20 days. (Offer of renewal is subject to Allstate's underwriting standards and practices.)

If we do not receive the Form within the next 20 days, any vehicle currently rated for farm use will instead be rated for pleasure use.

We strongly encourage you to complete and return the Form to us within the specified time. The premium for your upcoming policy renewal offer could be higher than it would be if we do not have the correct usage information.

Please contact your Allstate agent if you have any questions about this important matter or if you need help completing the enclosed Estimated Annual Mileage and Usage Verification Form.

Sincerely,

Allstate Insurance Company

F19

2005 TAX STATEMENT

SYLVIA S. ROMO, CPA, RTA, CTA
BEXAR COUNTY
TAX ASSESSOR-COLLECTOR
P.O. BOX 839950
SAN ANTONIO TX 78283-3950
TELEPHONE: (210) 335-2251
www.bexar.org/tax

REAL PROPERTY

ACCOUNT NUMBER
04708-100-1049

LEGAL DESCRIPTION:
CB 4708A BLK   LOT 1049

10/22/2005   **ACREAGE:**
00001.6100ACRES:

APPRAISED VALUE		CAP VALUE	HOMESTEAD VALUE	AGRL VALUE	NON-QUAL VALUE	ASSESSED VALUE
LAND 59,330	IMPR 262,950		322,280			322,280

TAXING UNIT	EXEMPTIONS					TAXABLE VALUE	TAX RATE	TAX AMOUNT
	HOM	DAV						
FLOOD FUND	3,000	5,000	0	0	0	314,280	.0127190	39.97
ALAMO COMM COLLEGE	0	5,000	0	0	0	317,280	.1070500	339.65
HOSPITAL DISTRICT	0	5,000	0	0	0	317,280	.2438690	773.75
BEXAR COUNTY	0	5,000	0	0	0	317,280	.3184710	1,010.44
S A RIVER AUTHORIT	5,000	5,000	0	0	0	312,280	.0164250	51.29
TOTAL TAXES THIS STATEMENT								$2,215.10

IF TAXES ARE TO BE PAID FROM MORTGAGE ESCROW PLEASE DO NOT PAY FROM THIS STATEMENT.

ASSESSMENT RATIO FOR ALL UNITS IS 100%.   SEE BACK OF STATEMENT FOR ADDITIONAL INFORMATION ABOUT YOUR TAXES.

(Please detach and return this stub with your payment)

2005 PAYMENT AMOUNTS

10/22/2005

SPLIT PAYMENT AMOUNT	1,107.55
QUARTER PAYMENT - IF QUALIFIED	
TOTAL CURRENT TAX DUE	2,215.10

DELINQUENCY DATE:
FEBRUARY 1, 2006

AMOUNT PAID:

MC/FID:

MAKE CHECK PAYABLE TO:
SYLVIA S. ROMO, CPA, RTA, CTA
BEXAR COUNTY
TAX ASSESSOR-COLLECTOR
P.O. BOX 839950
SAN ANTONIO TX 78283-3950

.5-4435

466

Name_____

Regarding the ALLSTATE letter:

(1) What does "premium" mean?

(2) Locate the statement: We will consider the information you provide on the Form before we calculate the premium for your upcoming policy renewal offer if we receive the completed form from you in the next 20 days." What does this statement imply will happen if Allstate does not receive the form in the next 20 days?

(3) Locate the statement: "The premium for your upcoming policy renewal offer could be higher than it would be if we do not have the correct usage information." If this statement is true, which of these scenarios could happen:

    (a) The premium goes down.

    (b) The premium goes up.

    (c) The premium remains the same.

(4) Write a sentence using the three words, "insure," "ensure," and "assure" so that their different nuances are illustrated.

****

Regarding the 2005 Tax Statement:

(1) What does "exemption" mean?

(2) What do "HOM" and "DAV" mean?

(3) How much does the homeowner save in taxes as a result of the DAV exemption?

(4) Refer to the column labeled "TAXING UNIT." What taxation authority appears conspicuous in its absence from this list? What could be the reason for this apparent omission?

In 1945 George Polya published the book, *How to Solve It,* which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. In this text he identifies four basic principles.

Polya's First Principle: Understand the Problem

This seems so obvious that it is often not even mentioned, yet students are often stymied in their efforts to solve a problem simply because they don't understand it fully, or even in part. Polya taught teachers to ask students questions such as:

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

Polya's Second Principle: Devise a plan

Polya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best acquired by solving many problems. You will find choosing a strategy increasingly easy. Here is a partial list of strategies:

• Guess and check	• Look for a pattern
• Make an orderly list	• Draw a picture
• Eliminate possibilities	• Solve a simpler problem
• Use symmetry	• Use a model
• Consider special cases	• Work backward
• Use direct reasoning	• Use a formula
• Solve an equation	• Be ingenious

Polya's third Principle: Carry out the plan

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work, discard it and choose another. Don't be misled; this is how mathematics is done, even by professionals.

Polya's Fourth Principle: Look back.

Much can be gained by taking the time to reflect on what you have done, what worked, and what didn't. Doing this will help you choose strategies to solve future problems.

Here's the history of this activity.

On September 21, 2006 the "Ripley's Believe It or Not" challenge in the local paper was to divide the region below into four mutually exclusive, congruent sections so that each section contains one of the four dots.
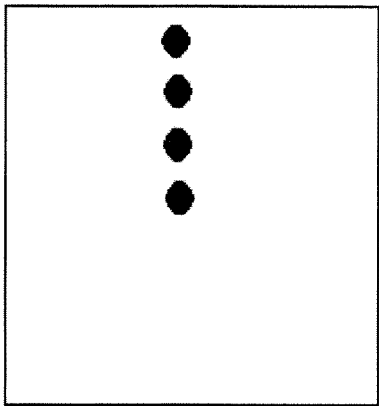
In solving this, I realized that one could employ George Polya's problem-solving principles, which I've included at the end of this exercise.

Here's a suggestion. Print out the following and give your students, your child, yourself—or anyone else you want to engage—one page at a time and see how many pages they need before they figure it out.

Enjoy!

The square country of Squaresylvania wants to split into four separate countries, Blue, Green, Red, and Purple. Unfortunately, the oil fields, gold mines, water wells, and silver mines are all located in the southeast quadrant as indicated on the map below.

O
G
W
S
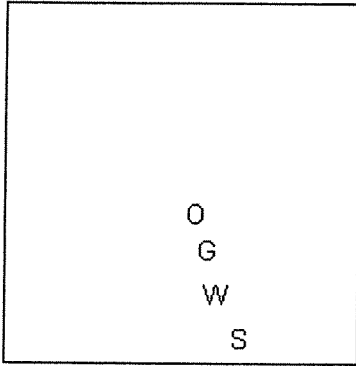
The people want each of the four new countries to get one of the natural resources. They also want all four of the new countries to consist of continuous regions of exactly the same size and shape!

After much frustrating work, all the great thinkers in the land have concluded that such a division is not possible. Well, they're wrong. ☺ Let's give it a shot! Before you go on to the next page, here are a few maps for you to try out some ideas.

O
G
W
S

O
G
W
S

O
G
W
S

O
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W
S

O
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W
S

O
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G
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S

O
G
W
S

It's not easy, is it? Each of the following pages provides a hint.

O
G
W
S

O
G
W
S

O
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W
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S

O
G
W
S

Maybe we can get a partial solution by dividing *part* of the country into four congruent pieces. For example, let's divide the bottom half into four equal parts. Want to give it another shot before you go on to the next page?

	O		O		O
	G		G		G
	W		W		W
S		S		S	
	O		O		O
	G		G		G
	W		W		W
S		S		S	
	O		O		O
	G		G		G
	W		W		W
S		S		S	
	O		O		O
	G		G		G
	W		W		W
S		S		S	

Perhaps adding the diagonals will help. Notice any symmetry?
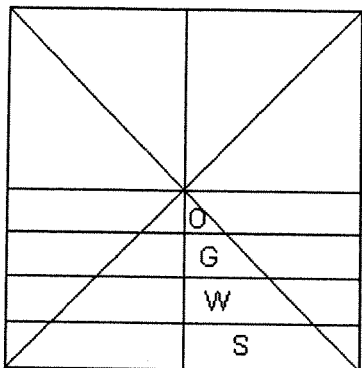
Want to try again before going on to the next page?

54 J
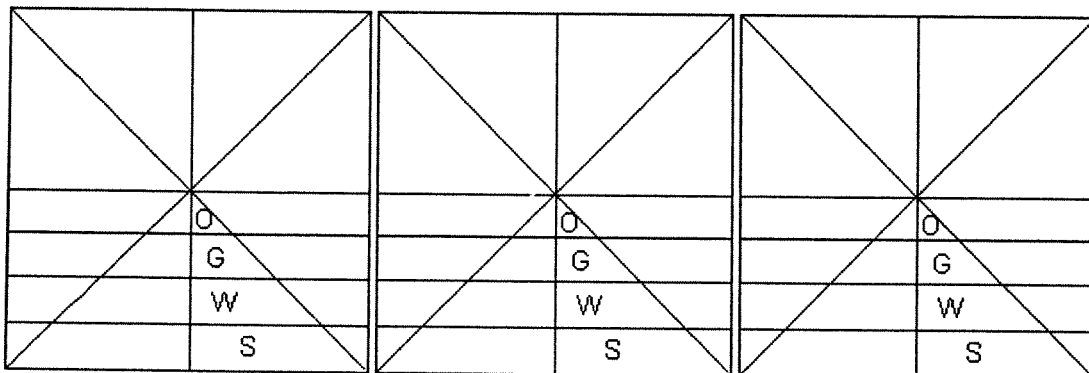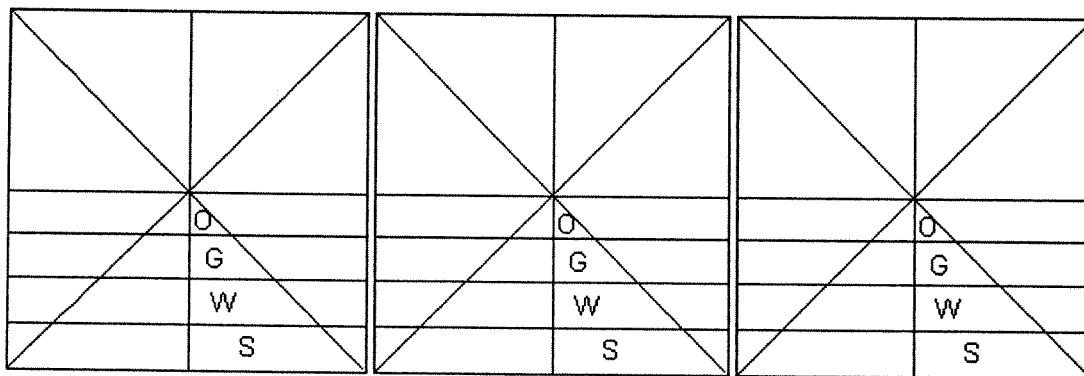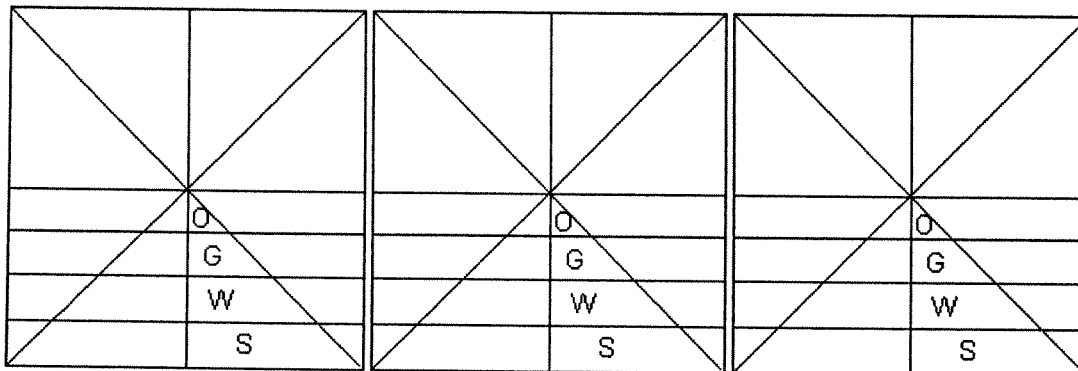
Okay, maybe it would help to erase a little. Any new ideas occur to you before you go on?
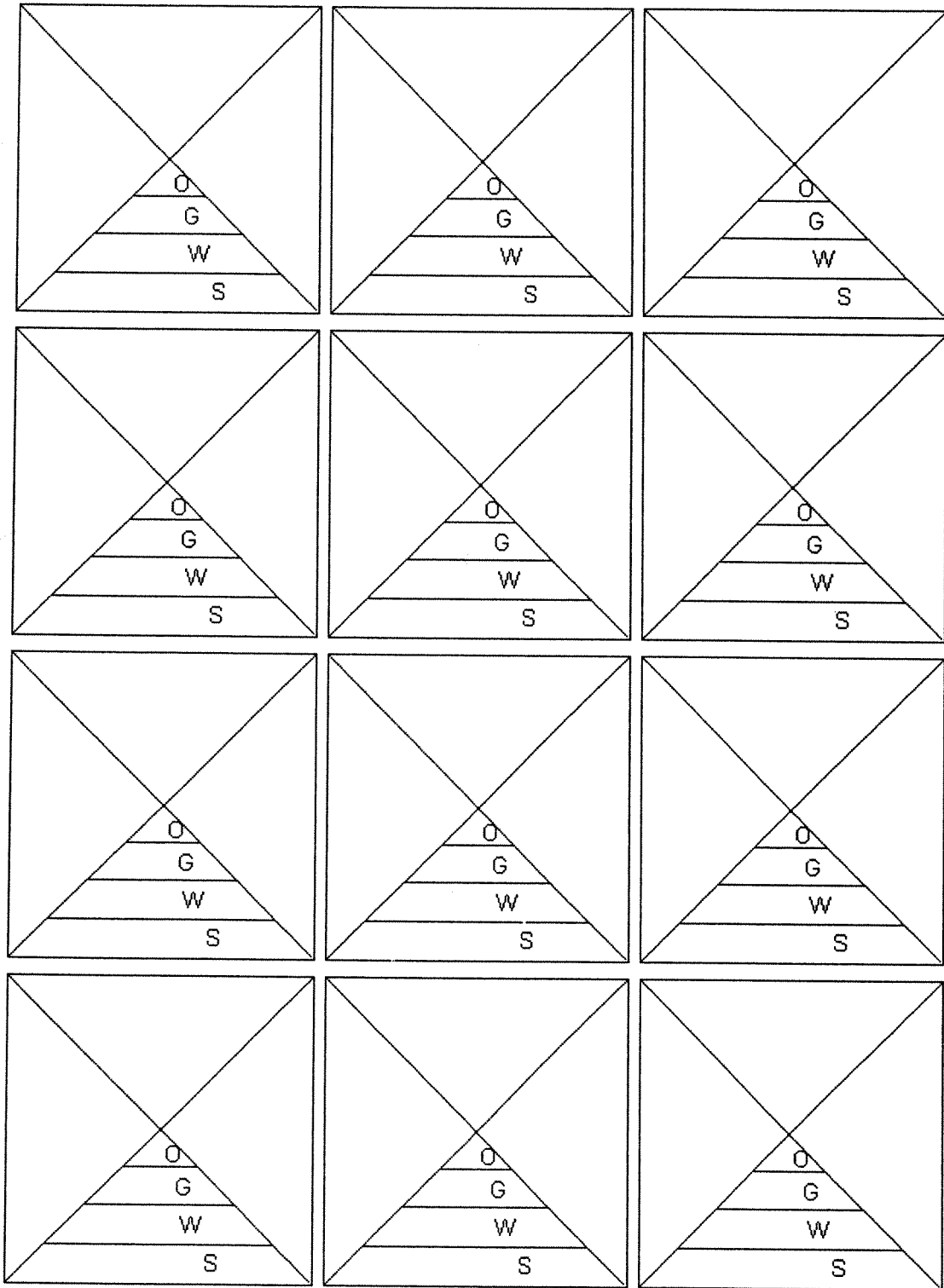
54K

Maybe it would help to divide the north, east, and west triangles to look just like the south triangle.
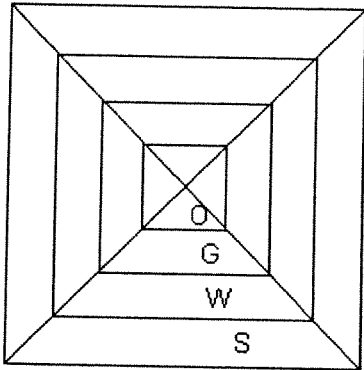
Do you see the four concentric squares? Does this give you an idea about another way we could have started this process and gotten here faster?

Want to try out some new ideas before looking at the next page?
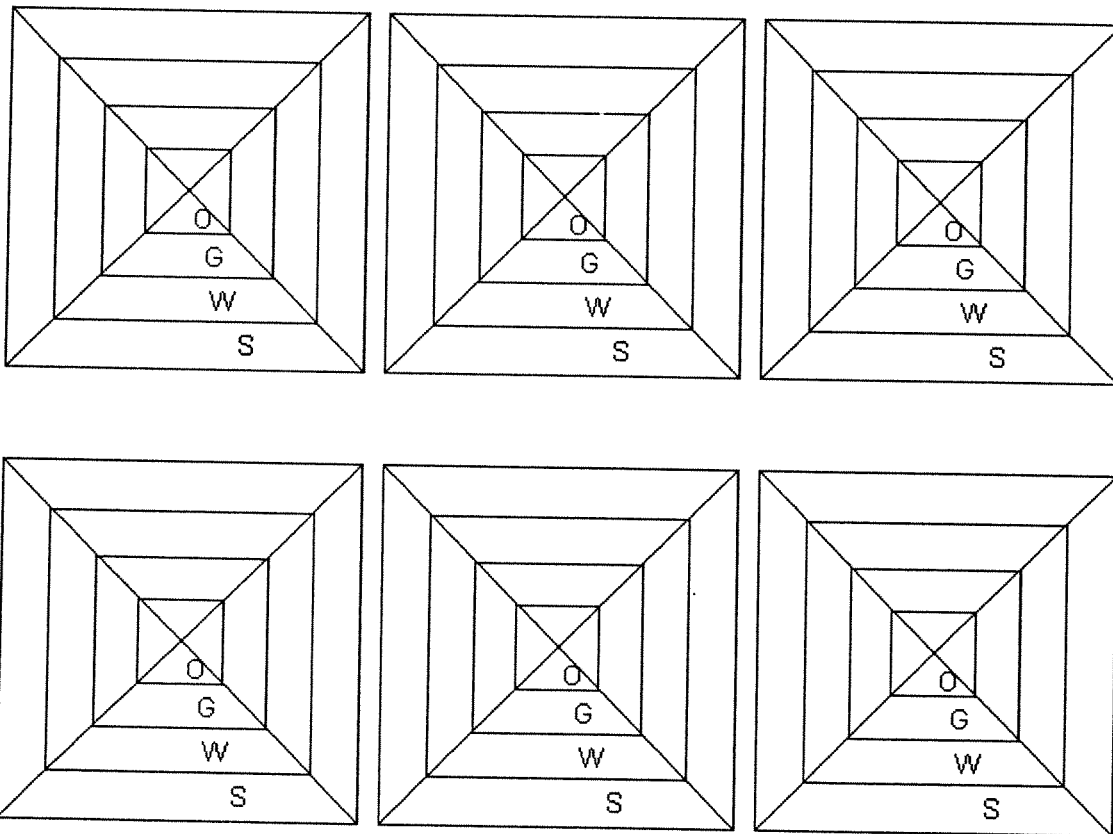
Notice that each of the four large triangles consists of four shapes, namely, one small triangle and three trapezoids.
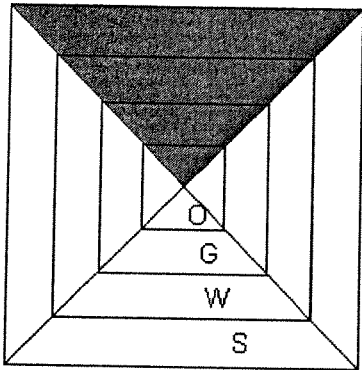
Without going into detail, what general strategy could you use to ensure that the four countries would have the same area? Want to give it a try?
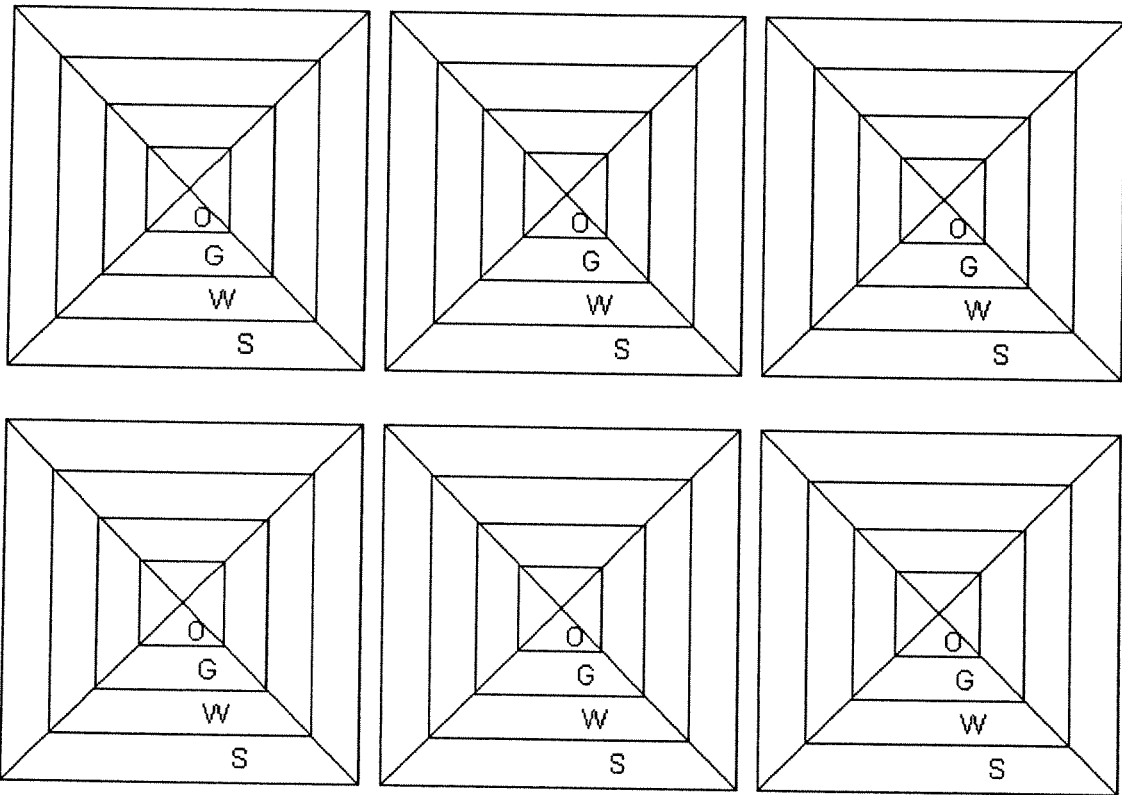
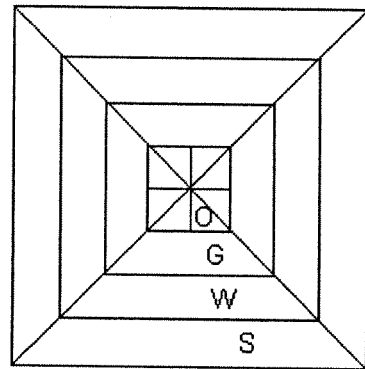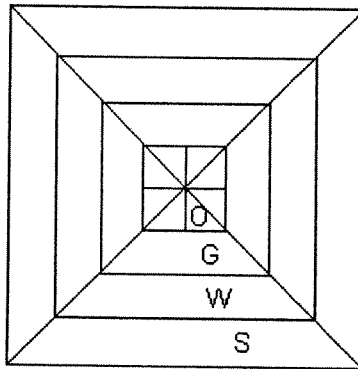Would you like a hint before you go to the next page? Here's one.
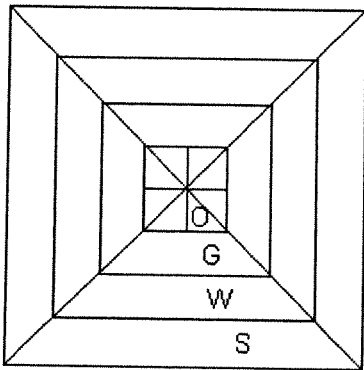
Let's try this.
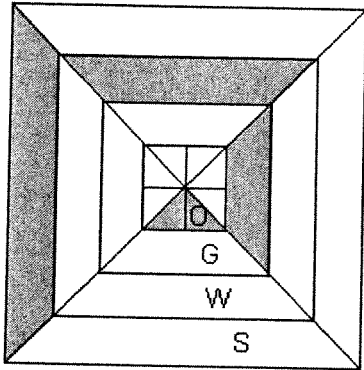
Now, can you do a little modifying so that the shaded areas are connected in more than one point?
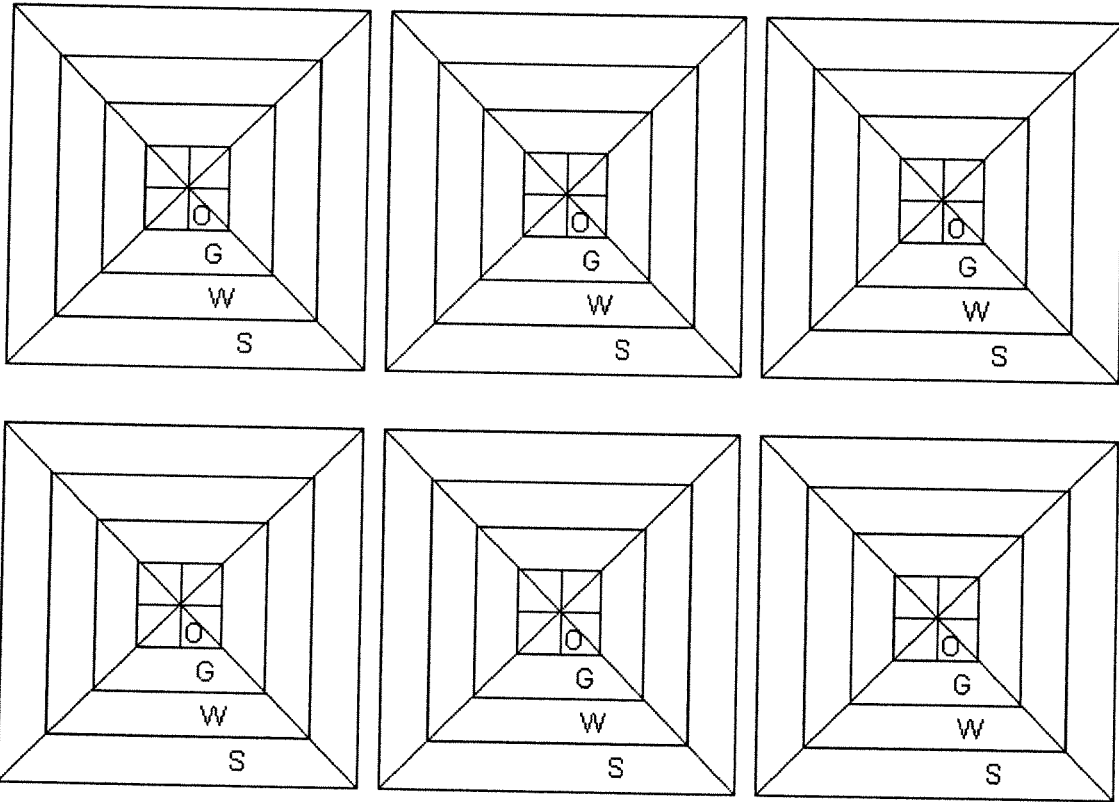
See next page for one possibility.

54N
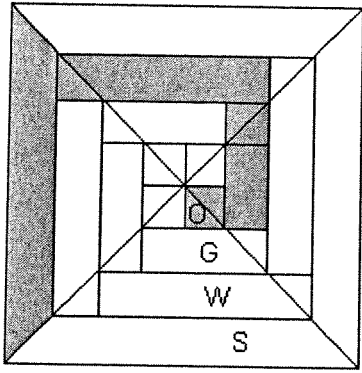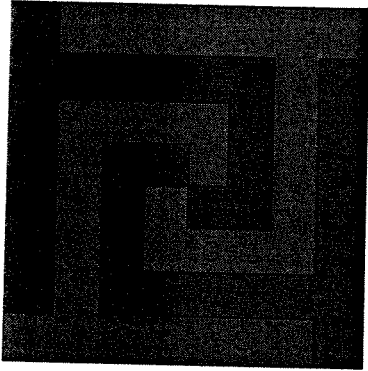
Can you now finish the job before going to one possible solution on next page?
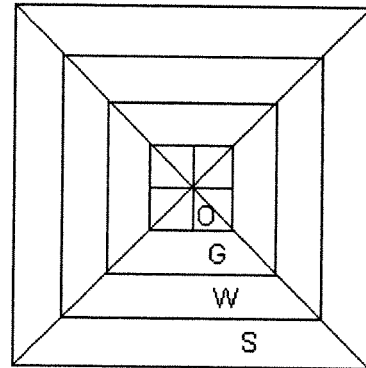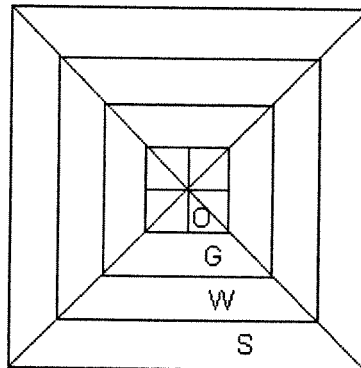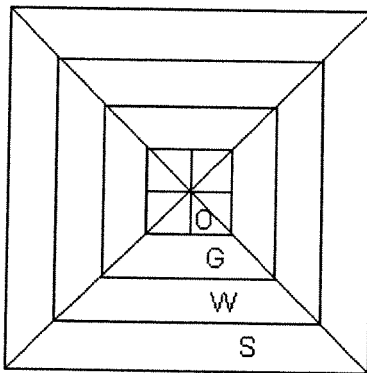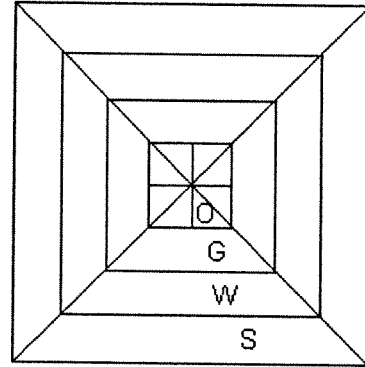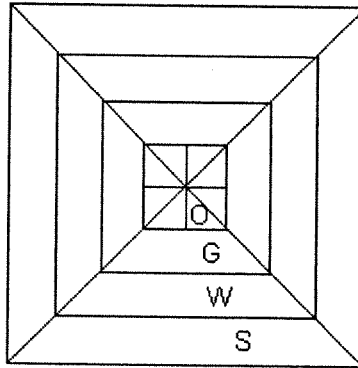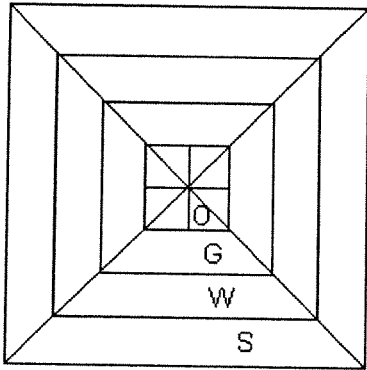
Do you see some other solutions, perhaps just a little different from this one?

Can you use this exercise to propose a solution to the war in Iraq? ☺

See the next page for some of George Polya's classic problem-solving strategies.