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Symmetry

Algebraic Test of Symmetry

***x-axis**:* If replacing y with $-y$ produces an equivalent equation, then the graph is *symmetric with respect to the x-axis.*

***y-axis**:* If replacing x with $-x$ produces an equivalent equation, then the graph is *symmetric with respect to the y-axis.*

***Origin**:* If replacing x with $-x$ and y with $-y$ produces an equivalent equation, then the graph is *symmetric with respect to the origin.*

Even and Odd Functions

If the graph of a function f is symmetric with respect to the y-axis, we say that it is an **even function**. That is, for each x in the domain of f, $f(x) = f(-x)$.

If the graph of a function f is symmetric with respect to the origin, we say that it is an **odd function.** That is, for each x in the domain of f, $f(-x) = -f(x)$.

Transformations

Vertical Translation: y = f(x) ± b

For $b > 0$,

the graph of $y = f(x) + b$ is the graph of $y = f(x)$ shifted *up b* units;
the graph of $y = f(x) - b$ is the graph of $y = f(x)$ shifted *down b* units.

Horizontal Translation: y = f(x ± d)

For $d > 0$,

the graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted *right d* units;

the graph of $y = f(x + d)$ is the graph of $y = f(x)$ shifted *left d* units.

Reflections

Across the x-axis: The graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the x-axis.

Across the y-axis: The graph of $y = f(-x)$ is the reflection of the graph of $y = f(x)$ across the y-axis.

Vertical Stretching and Shrinking: y = a f(x)

The graph of $y = a f(x)$ can be obtained from the graph of $y = f(x)$ by

stretching vertically for $|a| > 1$, or
shrinking vertically for $0 < |a| < 1$

For $a < 0$, the graph is also reflected across the x-axis.

Horizontal Stretching or Shrinking: y = f(cx)

The graph of $y = f(cx)$ can be obtained from the graph of $y = f(x)$ by

shrinking horizontally for $|c| > 1$, or
stretching horizontally for $0 < |c| < 1$.

For $c < 0$, the graph is also reflected across the y-axis.

Quadratic Formula

The solutions of $ax^2 + bx + c = 0, a \neq 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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The Vertex of a Parabola

The **vertex** of the graph of $f(x) = ax^2 + bx + c$ is

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

We calculate the x-coordinate — We substitute to find the y-coordinate

The Algebra of Functions

The Sums, Differences, Products, and Quotients of Functions

If f and g are functions and x is the domain of each function, then

$(f+g)(x) = f(x) + g(x)$
$(f-g)(x) = f(x) - g(x)$
$(fg)(x) = f(x) \cdot g(x)$
$(f/g)(x) = f(x)/g(x)$, provided $g(x) \neq 0$

Composition of Functions

The **composition function** $f \circ g$, the **composition** of f and g, is defined as

$$(f \circ g)(x) = f(g(x)),$$

where x is in the domain of g and $g(x)$ is in the domain of f.

One-to-One Functions

A function f is **one-to-one** if different inputs have different outputs—that is,

if $a \neq b$, then $f(a) \neq f(b)$

Or a function f is **one-to-one** if when the outputs are the same, the inputs are the same—that is,

if $f(a) = f(b)$, then $a = b$

Horizontal-Line Test

If it is possible for a horizontal line to intersect the graph of a function more than once, then the function is *not* one-to-one and its inverse is *not* a function.

Obtaining a Formula for an Inverse

If a function f is one-to-one, a formula for its inverse can generally be found as follows:

1. Replace $f(x)$ with y.
2. Interchange x and y.
3. Solve for y.
4. Replace y with $f^{-1}(x)$.

Exponential and Logarithmic Functions

The function $f(x) = a^x$, where x is a real number, $a > 0$ and $a \neq 1$, is called the **exponential function,** base a.

We define $y = \log_a x$ as that number y such that $x = a^y$, where $x > 0$ and a is a positive constant other than 1.

Summary of the Properties of Logarithms

Product Rule: $\log_a MN = \log_a M + \log_a N$
Power Rule: $\log_a M^p = p \cdot \log_a M$
Quotient Rule: $\log_a \frac{M}{N} = \log_a M - \log_a N$
Change-of-Base: $\log_b M = \frac{\log M}{\log b}$
Formula

Other Properties:

$\log_a a = 1$ $\quad$ $\log_a 1 = 0$

$\log_a a^x = x$ $\quad$ $a^{\log_a x} = x$

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Solving Exponential and Logarithmic Equations

Base-Exponent Property

For any $a > 0$, $a \neq 1$,

$a^x = a^y \leftrightarrow x = y$

Property of Logarithmic Equality

For any $M > 0, N > 0, a > 0$, and $a \neq 1$,

$$\log_a M = \log_a N \leftrightarrow M = N$$

A Logarithm is an Exponent

$$\log_a x = y \leftrightarrow x = a^y$$

Polynomial Functions

Even and Odd Multiplicity

If $(x-c)^k, k \geq 1$, is a factor of a polynomial function $P(x)$ and $(x-c)^{k+1}$ is not a factor of $P(x)$ and :

- k is odd, then the graph crosses the x-axis at $(c, 0)$;
- k is even, then the graph is tangent to the x-axis at $(c, 0)$

The Intermediate Value Theorem

For any polynomial function $P(x)$with real coefficients, suppose that for $a \neq b$, $P(a)$ and $P(b)$ are of opposite signs. Then the function has a real zero between a and b.

The Remainder Theorem

If a number c is substituted for x in the polynomial $f(x)$, then the result $f(c)$ is the remainder that would be obtained by dividing $f(x)$ by $x - c$. That is, if $f(x) = (x-c) \cdot Q(x) + R$, then $f(c) = R$.

The Factor Theorem

For a polynomial $f(x)$, if $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

The Fundamental Theorem of Algebra

Every polynomial function of degree n, with $n \geq 1$, has at least one zero in the system of complex numbers.

Nonreal Zeros: $\boldsymbol{a + bi}$ and $\boldsymbol{a - bi, b \neq 0}$

If a complex number $a + bi, b \neq 0$, is a zero of a polynomial function $f(x)$ with real coefficients, then its conjugate, $a - bi$, is a also a zero.

Irrational Zeros: $\boldsymbol{a + c\sqrt{b}}$ and $\boldsymbol{a - c\sqrt{b}}$, $\boldsymbol{b}$ is not a perfect square

If $a + c\sqrt{b}$ and $a - c\sqrt{b}$, b is not a perfect square, is a zero of a polynomial function $f(x)$ with rational coefficients, then its conjugate, $a - c\sqrt{b}$, is also a zero. For example, if $-3 + 5\sqrt{2}$ is a zero of a polynomial function $f(x)$, with rational coefficients, then its conjugate, $-3 - 5\sqrt{2}$, is also a zero.

The Rational Zeros Theorem

Let $P(x) = a_n x^n + a_{n-1} x^n + \cdots + a_1 x + a_0$, where all the coefficients are integers. Consider a rational number denoted by p/q, where p and q are relatively prime. If p/q is a zero of $P(x)$, then p is a factor of a_0 and q is a factor of a_n.

Ex. $3x^4 - 11x^3 + 10x - 4$

$$\frac{Possibilities\ for\ p\ (a_0)}{Possibilities\ for\ q\ (a_n)}: \qquad \frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3}$$

$Possibilities\ for\ p/q$:

$1, -1, 2, -2, 4, -4, \frac{1}{3}, \frac{-1}{3}, \frac{2}{3}, \frac{-2}{3}, \frac{4}{3}, \frac{-4}{3}$

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Descartes' Rule of Signs

Let $P(x)$, written in descending or ascending order, be a polynomial function with real coefficients and a nonzero constant term. The number of positive real zeros of $P(x)$ is either:

1. The same as the number of variations of sign in $P(x)$, or
2. Less than the number of variations of sign in $P(x)$ by a positive even integer.

The number of negative real zeros of $P(x)$ is either:

3. The same as the number of variations of sign in $P(-x)$, or
4. Less than the number of variations of sign in $P(-x)$ by a positive even integer.

A zero of multiplicity *m* must be counted *m* times.