Radical Expressions

Square roots of perfect squares

- A perfect square is an integer multiplied by itself.

Examples of perfect squares:

- 1, 4, 9, 16, 25, and 36 are examples of perfect squares because

$1^2 = 1$ $\quad$ $4^2 = 16$

$2^2 = 4$ $\quad$ $5^2 = 25$

$3^2 = 9$ $\quad$ $6^2 = 36$

Note that squaring the negative integers results in the same list of numbers

$(-1)^2 = 1$ $\quad$ $(-4)^2 = 16$

$(-2)^2 = 4$ $\quad$ $(-5)^2 = 25$

$(-3)^2 = 9$ $\quad$ $(-6)^2 = 36$

The square root of a number (X) is a number (a) which when multiplied by itself (a^2) gives us X

$\sqrt{X} = a$ if $a^2 = X$

Examples of square roots:

Simplify $\sqrt{49}$

$7^2 = 49$ so $\sqrt{49} = 7$

Simplify $\sqrt{25} + \sqrt{81}$

$5^2 = 25$ and $9^2 = 81$

so $\sqrt{25} + \sqrt{81} = 5 + 9$

$= 14$

Square roots of whole numbers:

When taking the square root of a number that is not a perfect square, the answer can only be estimated.

Examples:

Between what two whole numbers is the value of $\sqrt{41}$?

Since $6^2 = 36$ and $7^2 = 49$ then the $\sqrt{41}$ must be between 6 and 7.

Between what two whole numbers is the value of $\sqrt{14}$?

Since $3^2 = 9$ and $4^2 = 16$, then the $\sqrt{14}$ must be between 3 and 4.

Approximating the square root of a number to the nearest tenth:

Step 1 – get the difference between the two perfect squares closest to the number, one greater than and one less than the number.

Step 2 – get the difference between the number we are trying to take the square root of and the perfect square that is less than the number.

Step 3 – now divide the difference between the two perfect squares (found in step 1) by the difference between the perfect square and the number (found in step 2).

Step 4 – add the answer from step 3 to the square root of the smaller perfect square.

Example: Approximate $\sqrt{59}$ to the nearest tenth.

The $\sqrt{59}$ is between the $\sqrt{49}$ (the closest perfect square less than 59)
and $\sqrt{64}$ (the closest perfect square greater than 59)

Step 1 – the difference between the perfect squares is 64 – 49 = 15

Step 2 – the difference between the number and the smallest perfect square is 59 – 49 = 10

Step 3 – divide the two difference $10/15 \approx 0.7$

Step 4 – add our answer from step 3 to the square root of the smallest perfect square 7 + 0.7 = 7.

So $\sqrt{59} \approx 7.7$

Applications and formulas:

Example: Find the range of a submarine periscope that is 9ft above the surface of the water. Use the formula $R = 1.4\sqrt{h}$, where R is the range in miles and h is the height in feet of the periscope above the surface of the water. Round to the nearest hundredth.

We start with our equation $R = 1.4\sqrt{h}$

and replace the value of 9 for h $R = 1.4\sqrt{9}$

Now we take the square root of 9 $R = 1.4(3)$

Performing the multiplication gives us $R = 4.2$

So now we know that the range is 4.20 miles.