Multiplication and Division of Integers

Multiplication of Integers

When 5 is multiplied by a sequence of decreasing integers, each product decreases by 5.

$5(3) = 15$
$5(2) = 10$
$5(1) = 5$
$5(0) = 0$

The pattern developed can be continued so that 5 is multiplied by a sequence of negative numbers. To maintain the pattern of decreasing by 5, the resulting products must be negative.

$5(-1) = -5$
$5(-2) = -10$
$5(-3) = -15$
$5(-4) = -20$

This example illustrates that the product of a positive number and a negative number is negative.

When –5 is multiplied by a sequence of decreasing integers, each product increases by 5.

$-5(3) = -15$
$-5(2) = -10$
$-5(1) = -5$
$-5(0) = 0$

The pattern developed can be continued so that –5 is multiplied by a sequence of negative numbers. to maintain the pattern of increasing by 5, the resulting products must be positive.

$-5(-1) = 5$
$-5(-2) = 10$
$-5(-3) = 15$
$-5(-4) = 20$

This example illustrates that the product of two negative numbers is positive.

The proceeding pattern is summarized in the following rule for multiplying integers.

Rule for Multiplying Two Integers:

TO MULTIPLY INTEGERS WITH THE SAME SIGN, multiply the absolute value of the factors. The product is positive.

TO MULTIPLY INTEGERS WITH DIFFERENT SIGNS, multiply the absolute value of the factors. The product is negative.

Multiply: -9(12) = ?

The signs are different. The product is negative. -9(12) = -108

Multiply: (-6)(-15) = ?

The signs are the same. The product is positive. (-6)(-15) = 90

The properties of multiplication presented in Chapter 1 hold true for integers as well as whole numbers. These properties are repeated below.

The Multiplication Property of Zero

a * 0 = 0 or 0 * a = 0

The Multiplication Property of One

a * 1 = a or 1 * a = a

The Commutative Property of Multiplication

a * b = b* a

The Associative Property of Multiplication

(a * b) * c = a * (b * c)

Multiply: 2(-3)(-5)(-7)

2(-3)(-5)(-7)

Multiply the first two numbers. = -6(-5)(-7)

Then multiply the product by the third number. = 30(-7)

Continue until all the numbers have been multiplied = -210

By the Multiplication Property of One, 1 * 6 = 6 and 1 * x = x. Applying the rules for multiplication, we can extend this to 1 * -6 = -6 and 1 * -x = -x.

Evaluate: – ab when a = –2 and b = – 9.

Replace a with –2 and b with – 9.	– (– 2)(– 9)
Simplify – (–2)	= 2(– 9)
Multiply.	= – 18

Example:

Is – 4 a solution of the equation 5x = –20?	5x = – 20
Replace x with – 4 and then simplify.	5(– 4) = – 20
The results are equal.	–20 = –20

Yes, – 4 is a solution of the equation.

Division of Integers:

For every division problem, there is a related Multiplication problem.

Division 8/2 = 4 Related multiplication: 4(2) = 8

This fact can used to illustrate a rule for dividing Integers.

12/3 = 4 because 4(3) = 12 and

-12/-3 = 4 because 4(-3) = -12

These two division examples suggest that the quotient of two numbers with the same sign is positive.

Now consider these two examples.

12/–3 = – 4 because – 4(–3) = 12 and

–12/3 = – 4 because – 4(3) = –12

These two division examples suggest that the quotient of two numbers with different signs is negative. This property is summarized next.

Take Note:

Recall that the fraction bar can be read "divided by."

Therefore, 8/2 can be read as "8 divided by 2."

Rule for Dividing Two Integers:

TO DIVIDE TWO NUMBERS WITH THE SAME SIGN, divide the absolute values of the numbers. The quotient is positive.

TO DIVIDE TWO NUMBERS WITH THE DIFFERENT SIGNS, divide the absolute values of the numbers. The quotient is negative.

Note from this rule that 12/–3 and –12/3 are equal to – 4.

If a and b are integers (b is not equal 0), then a/-b = -a/b.

Divide: -36/9 = ?

The signs are different. The quotient is negative. -36/9 = -4

Divide: (-105)/(-5) = ?

The signs are the same. The quotient is positive. (-105)/(-5) = 21

Division Properties of Zero and One:

If a is not equal to 0, then 0/a = 0.
If a is not equal to 0, then a/a = 1.
If b is not equal to 0, then a/b = a * 1/b
a/1 = a
a/0 is undefined

Evaluate a / (-b) when a = –28 and b = – 4	a/(-b)
Replace a with –28 and b with – 4	– 28 ÷ (– (– 4))
Simplify – (– 4).	–28 ÷ 4
Divide.	–7