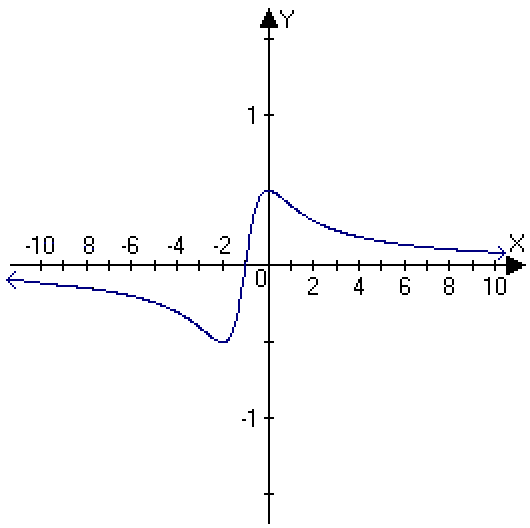
Review Exercise Set 12

Exercise 1: Find the open intervals on the graph where the given function is increasing or decreasing.



Increasing:

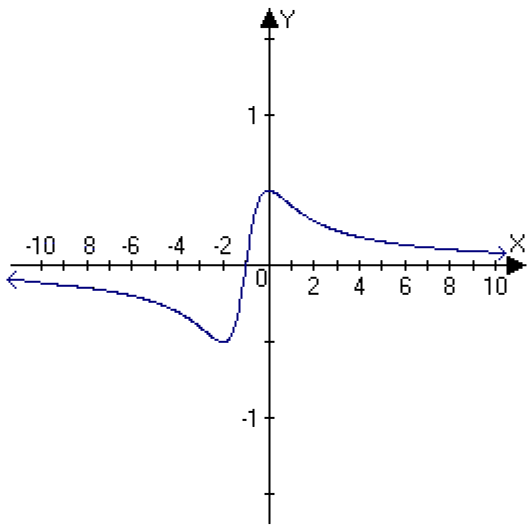
Decreasing:

Exercise 2: Find the open intervals where the given function is increasing or decreasing.

$y = x^3 + x^2 - 8x + 5$

Exercise 3: Find the locations and values of all relative extrema for the function in the graph below.

$f(x) = (x + 1) / (x^2 + 2x + 2)$



Exercise 4: Find the values of any relative extrema for the given function.

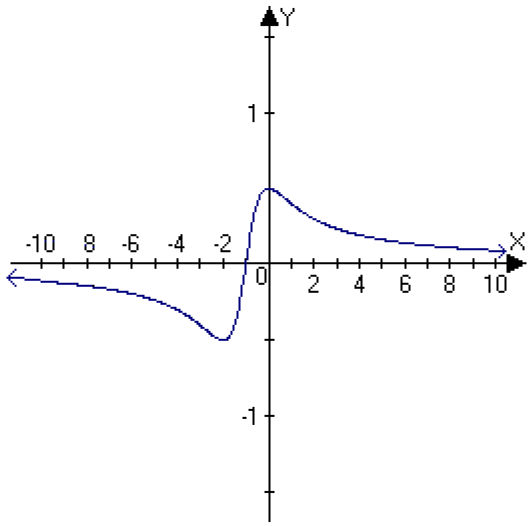
$y = x^{2/3} (x^2 - 4)$

Exercise 5: Find the values of any relative extrema for the given function.

$$f(x) = x\,(4 - x^2)^{1/2}$$

Review Exercise Set 12 Answer Key

Exercise 1: Find the open intervals on the graph where the given function is increasing or decreasing.



Increasing: $(-2, 0)$

Decreasing: $(-\infty, -2) \cup (0, \infty)$

Exercise 2: Find the open intervals where the given function is increasing or decreasing.

$y = x^3 + x^2 - 8x + 5$

Find the first derivative

$y = x^3 + x^2 - 8x + 5$

$y' = 3x^2 + 2x - 8$

Find the critical numbers

$0 = 3x^2 + 2x - 8$

$0 = (3x - 4)(x + 2)$

$3x - 4 = 0$ or $x + 2 = 0$

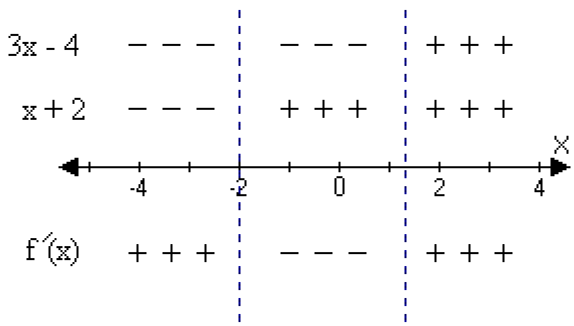
$x = \frac{4}{3}$ or $x = -2$

Exercise 2 (Continued):

Test each interval to determine the sign of the derivative

Test points: -4, 0, 2

	$x=-4$	$x=0$	$x=2$
$3x-4$	$-12-4=-16$	$0-4=-4$	$6-4=2$
$x+2$	$-4+2=-2$	$0+2=2$	$2+2=4$
$f'(x)$	$(-16)(-2)=32$	$(-4)(2)=-8$	$(2)(4)=8$

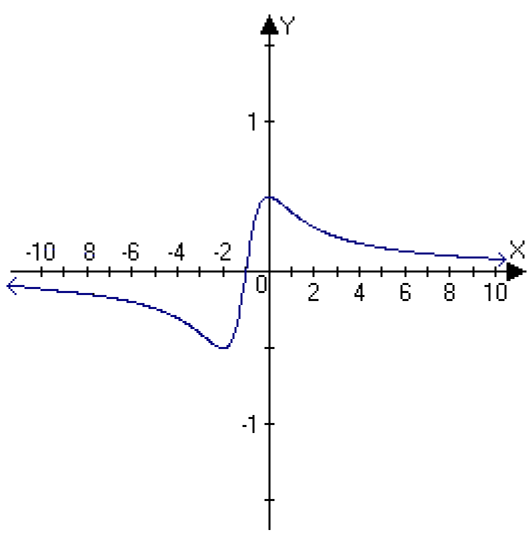


Increasing: $(-\infty,-2)\cup\left(\frac{4}{3},\infty\right)$

Decreasing: $\left(-2,\frac{4}{3}\right)$

Exercise 3: Find the locations and values of all relative extrema for the function in the graph below.

f(x) = (x + 1) / (x^2 + 2x + 2)



Exercise 3 (Continued):

Locate the x-coordinate of the relative extrema on the graph

From the graph the relative minimum will occur when x = -2 and the relative maximum will occur when x = 0.

Evaluate the function at these x-values to find the y-coordinates of the points.

$$f(-2) = \frac{-2+1}{(-2)^2 + 2(-2) + 2}$$
$$= \frac{-1}{4-4+2}$$
$$= -\frac{1}{2}$$

relative minimum is at $\left(-2, -\frac{1}{2}\right)$

$$f(0) = \frac{0+1}{(0)^2 + 2(0) + 2}$$
$$= \frac{1}{2}$$

relative maximum is at $\left(0, \frac{1}{2}\right)$

Exercise 4: Find the values of any relative extrema for the given function.

$y = x^{2/3} (x^2 - 4)$

Find the derivative

$y' = x^{2/3} D_x(x^2 - 4) + (x^2 - 4)D_x(x^{2/3})$

$y' = x^{2/3} (2x) + (x^2 - 4)(\frac{2}{3} x^{-1/3})$

$y' = 2x^{5/3} + \frac{2}{3}x^{5/3} - \frac{8}{3}x^{-1/3}$

$y' = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3}$

Exercise 4 (Continued):

Find the critical numbers

$0 = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3}$

$0 = \frac{8}{3}x^{-1/3}(x^2 - 1)$

$0 = x^{-1/3}(x^2 - 1)$

Set each factor equal to zero and solve for x

$x^{-1/3} = 0$

$\frac{1}{\sqrt[3]{x}} = 0$

The fraction cannot be zero because the numerator is a constant but we can still get a critical number by setting the denominator equal to zero.

$\sqrt[3]{x} = 0$

$x = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

The critical numbers are -1, 0, and 1

Evaluate the function at the critical numbers

$y = x^{2/3}(x^2 - 4)$; when x = -1

$y = (-1)^{2/3}((-1)^2 - 4)$

$y = (1)(1 - 4)$

$y = -3$

$y = x^{2/3}(x^2 - 4)$; when x = 0

$y = (0)^{2/3}((0)^2 - 4)$

$y = (0)(0 - 4)$

$y = 0$

$y = x^{2/3}(x^2 - 4)$; when x = 1

$y = (1)^{2/3}((1)^2 - 4)$

$y = (1)(1 - 4)$

$y = -3$

The relative maximum would be at (0,0) and the relative minimum would be at (-1, -3) and (1, -3).

Exercise 5: Find the values of any relative extrema for the given function.

$f(x) = x (4 - x^2)^{1/2}$

Find the derivative

$f'(x) = x\ D_x(4 - x^2)^{1/2} + (4 - x^2)^{1/2} D_x(x)$

$f'(x) = x\ [\frac{1}{2}\ (4 - x^2)^{-1/2}(-2x)] + (4 - x^2)^{1/2}(1)$

$f'(x) = -x^2(4 - x^2)^{-1/2} + (4 - x^2)^{1/2}$

$f'(x) = (4 - x^2)^{-1/2}\ [-x^2 + (4 - x^2)]$

$f'(x) = (4 - x^2)^{-1/2}\ (-2x^2 + 4)$

Find the critical numbers

$0 = (4 - x^2)^{-1/2}\ (-2x^2 + 4)$

$(4 - x^2)^{-1/2} = 0$

$\frac{1}{\sqrt{4-x^2}} = 0$

The fraction cannot be zero so we will set the denominator equal to zero.

$\sqrt{4-x^2} = 0$

$4 - x^2 = 0$

$4 = x^2$

$\pm 2 = x$

$-2x^2 + 4 = 0$

$-2x^2 = -4$

$x^2 = 2$

$x = \pm\sqrt{2}$

The critical numbers are -2, $-\sqrt{2}$, $\sqrt{2}$, and 2.

Evaluate the function at the critical numbers

$f(x) = x (4 - x^2)^{1/2}$; when x = -2

$f(-2) = (-2)(4 - (-2)^2)^{1/2}$

$= (-2)(0)^{1/2}$

$= 0$

Exercise 5 (Continued):

$f(x) = x\,(4 - x^2)^{1/2}$; when $x = -\sqrt{2}$

$f(-\sqrt{2}\,) = (-\sqrt{2}\,)(4 - (-\sqrt{2}\,)^2)^{1/2}$

$= (-\sqrt{2}\,)(2)^{1/2}$

$= -2$

$f(x) = x\,(4 - x^2)^{1/2}$; when $x = \sqrt{2}$

$f(\sqrt{2}\,) = (\sqrt{2}\,)(4 - (\sqrt{2}\,)^2)^{1/2}$

$= (\sqrt{2}\,)(2)^{1/2}$

$= 2$

$f(x) = x\,(4 - x^2)^{1/2}$; when $x = 2$

$f(2) = (2)(4 - (2)^2)^{1/2}$

$= (2)(0)^{1/2}$

$= 0$

The relative minimum would be at $(-\sqrt{2}\,, -2)$ and the relative maximum would be at $(\sqrt{2}\,, 2)$.