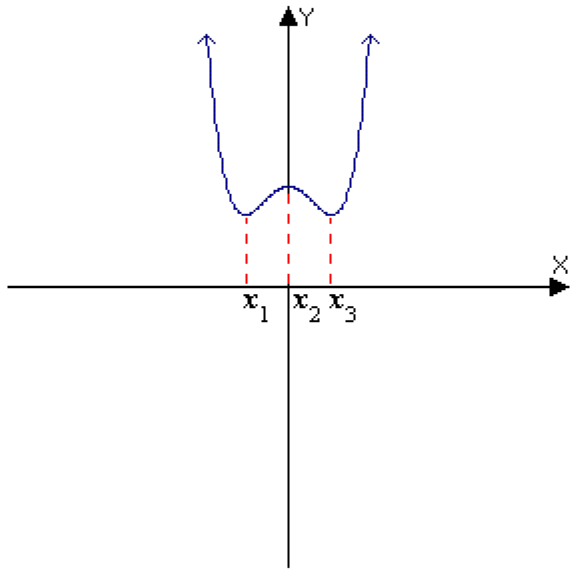
# Review Exercise Set 15

Exercise 1: Indicate the location of the absolute extrema (if any) for the function in the given graph.




Absolute minimum:

Absolute maximum:

Exercise 2: Find all absolute extrema within the specified closed interval for the function:

$g(x) = x^3 - x^2 + 8$ ; [-1, 1]

Exercise 3: Find all absolute extrema within the specified closed interval for the function:

$y = 3 / (x^2 + 2)$ ; [0, 4]

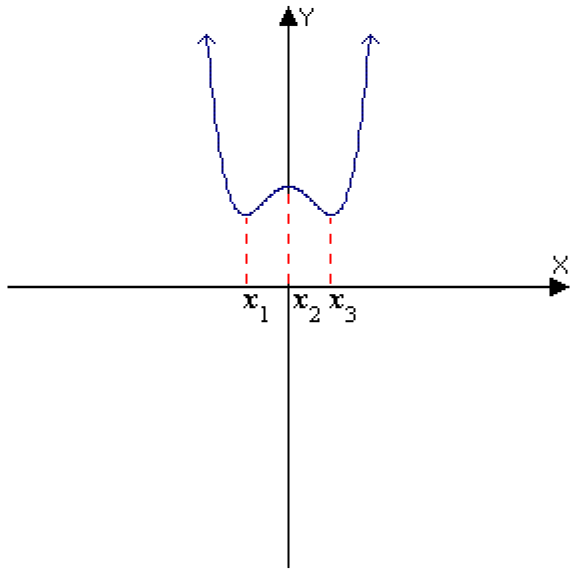
Exercise 4: Find all absolute extrema (if they exist) for the function:

$f(x) = x^4 - 4x^3 + 4x^2 + 4$

Exercise 5: If a manufacturing companies cost function is defined as $c(x) = 2000 + 96x + 4x^{3/2}$ (where x represents thousands of units), at what production level would the average cost be minimized?

Review Exercise Set 15 Answer Key

Exercise 1: Indicate the location of the absolute extrema (if any) for the function in the given graph.




Absolute minimum: $x_1$ and $x_3$

Absolute maximum: none, because the function increasing without bound as x approaches positive or negative infinity. $x_2$ is a relative maximum but not an absolute maximum.

Exercise 2: Find all absolute extrema within the specified closed interval for the function:

$g(x) = x^3 - x^2 + 8$ ; [-1, 1]

Find the first derivative

$g'(x) = 3x^2 - 2x$

Find the critical numbers of the first derivative

$0 = 3x^2 - 2x$
$0 = x(3x - 2)$
$x = 0$ or $3x - 2 = 0$
$x = 0$ or $x = \frac{2}{3}$

Exercise 2 (Continued):

Evaluate the function at the critical numbers and interval endpoints

$g(x) = x^3 - x^2 + 8;\ x = -1$
$g(-1) = (-1)^3 - (-1)^2 + 8$
$g(-1) = -1 - 1 + 8$
$g(-1) = 6$

$g(x) = x^3 - x^2 + 8;\ x = 0$
$g(0) = (0)^3 - (0)^2 + 8$
$g(0) = 0 - 0 + 8$
$g(0) = 8$

$g(x) = x^3 - x^2 + 8;\ x = \frac{2}{3}$
$g(\frac{2}{3}) = (\frac{2}{3})^3 - (\frac{2}{3})^2 + 8$
$g(\frac{2}{3}) = \frac{8}{27} - \frac{4}{9} + 8$
$g(\frac{2}{3}) = \frac{212}{27}$
$g(\frac{2}{3}) = 7\frac{23}{27}$

$g(x) = x^3 - x^2 + 8;\ x = 1$
$g(1) = (1)^3 - (1)^2 + 8$
$g(1) = 1 - 1 + 8$
$g(1) = 8$

Identify the absolute extrema

Absolute minimum: (-1, 6)

Absolute maximum: (0, 8) and (1, 8)

Exercise 3: Find all absolute extrema within the specified closed interval for the function:

$y = 3 / (x^2 + 2)$ ; [0, 4]

Find the first derivative

$y = 3(x^2 + 2)^{-1}$
$y' = -3(x^2 + 2)^{-2}(2x)$

$$y' = \frac{-6x}{\left(x^2 + 2\right)^2}$$

Exercise 3 (Continued):

Find the critical numbers of the first derivative

Set numerator equal to zero

$-6x = 0$
$x = 0$

Set denominator equal to zero

$(x^2 + 2)^2 = 0$
$x^2 + 2 = 0$
$x^2 = -2$

$x^2$ cannot be negative, so the denominator produces no critical numbers

Evaluate the function at the critical numbers and interval endpoints

$y = \frac{3}{x^2+2}$ ; x = 0

$y = \frac{3}{(0)^2+2}$

$= \frac{3}{2}$

$y = \frac{3}{x^2+2}$ ; x = 4

$y = \frac{3}{(4)^2+2}$

$= \frac{3}{18}$

Identify the absolute extrema

Absolute minimum: $(4, \frac{3}{18})$

Absolute maximum: $(0, \frac{3}{2})$

Exercise 4: Find all absolute extrema (if they exist) for the function:

$f(x) = x^4 - 4x^3 + 4x^2 + 4$

Find the first derivative

$f'(x) = 4x^3 - 12x^2 + 8x$

Exercise 4 (Continued):

Find the critical numbers of the first derivative

$0 = 4x(x^2 - 3x + 2)$
$0 = 4x(x - 1)(x - 2)$
$4x = 0$ or $x - 1 = 0$ or $x - 2 = 0$
$x = 0$ or $x = 1$ or $x = 2$

Evaluate the function at the critical numbers

$f(x) = x^4 - 4x^3 + 4x^2 + 4; x = 0$
$f(0) = (0)^4 - 4(0)^3 + 4(0)^2 + 4$
$f(0) = 4$

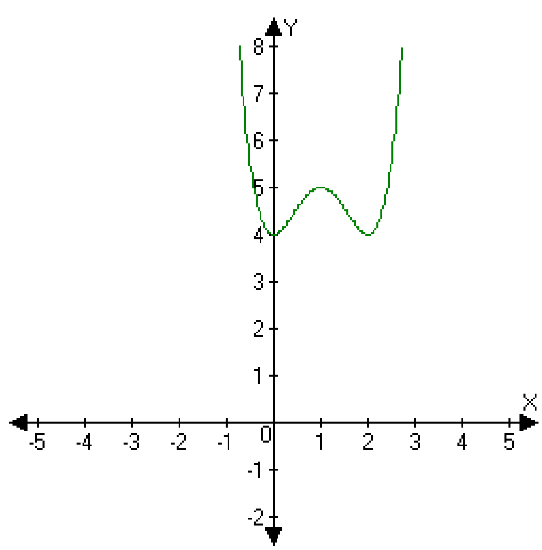
$f(x) = x^4 - 4x^3 + 4x^2 + 4; x = 1$
$f(1) = (1)^4 - 4(1)^3 + 4(1)^2 + 4$
$f(1) = 1 - 4 + 4 + 4$
$f(1) = 5$

$f(x) = x^4 - 4x^3 + 4x^2 + 4; x = 2$
$f(2) = (2)^4 - 4(2)^3 + 4(2)^2 + 4$
$f(2) = 16 - 32 + 16 + 4$
$f(2) = 4$

Identify the absolute extrema

Absolute minimum: (0, 4) and (2, 4)

Absolute maximum: none, because this is a function that increases without bound as x approaches positive and negative infinity.



Exercise 5: If a manufacturing companies cost function is defined as $c(x) = 2000 + 96x + 4x^{3/2}$ (where x represents thousands of units), at what production level would the average cost be minimized?

Find the average cost function

$$\overline{C(x)} = \frac{C(x)}{x}$$
$$= \frac{2000 + 96x + 4x^{3/2}}{x}$$
$$= \frac{2000}{x} + \frac{96x}{x} + \frac{4x^{3/2}}{x}$$
$$= 2000x^{-1} + 96 + 4x^{1/2}$$

Find the marginal average cost function

$$\overline{C'(x)} = -2000x^{-2} + 2x^{-1/2}$$

Find the critical numbers for the marginal average cost function

$0 = -2000x^{-2} + 2x^{-1/2}$
$0 = x^{-2}(-2000 + 2x^{3/2})$
$$0 = \frac{-2000 + 2x^{3/2}}{x^2}$$

Set numerator equal to zero

$0 = -2000 + 2x^{3/2}$
$2000 = 2x^{3/2}$
$1000 = x^{3/2}$
$1000^{2/3} = x$
$100 = x$

Set denominator equal to zero

$0 = x^2$
$0 = x$

The average cost function is not defined when x is zero so this cannot be a location for the absolute minimum.

Exercise 5 (Continued):

Evaluate the average cost function at the critical numbers

$$\overline{C(x)} = \frac{2000}{x} + 96 + 4x^{1/2}$$
$$\overline{C(100)} = \frac{2000}{100} + 96 + 4(100)^{1/2}$$
$$= 20 + 96 + 40$$
$$= 156$$

Identify the absolute minimum

Since x is in thousands of units, the average cost would be minimized when the production level is at 100,000 units.