Basic Rules of Differentiation

In the previous sections, you learned how to find the derivative of a function by using the formal definition of a derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Now that you know how to find the derivative with the use of limits, we will look at some rules that will simplify the process of finding the derivative. All of these rules can be proven by the use of the definition of the derivative and the rules of limits, but you only need to know when and how to apply the following derivative rules.

There are many different notations that can be used to represent the derivative of a function. Even though the notations may look very different they all represent the same thing – the derivative of a function. Below are some of the more common notations that you will come across:

$$f'(x); \quad y'; \quad \frac{dy}{dx}; \quad \frac{d}{dx}\left[f(x)\right]; \quad D_x\left[f(x)\right]; \quad D_x y$$

Even though “f” and “x” are the most common variables used to represent the name of a function and the independent variable other variables may be used. For instance, you may have functions named g(t); h(s); or even v(z). No matter what variables are used in the function the rules for finding the derivative will be applied in the same manner.

Lets begin by looking at the derivative of a constant function. A constant function is a horizontal line in for form of y = k or f(x) = k. The slope of a horizontal line is always equal to zero. Since the derivative of a function represents the slope of the function, the derivative of a constant function must be equal to its slope of zero. This gives you the first derivative rule – the Constant Rule.

Constant Rule

If f(x) = k, where k is any real number, then the derivative is equal to zero.

$$f'(x) = \frac{d}{dx}(k) = 0$$

Example 1: Find the derivative of the constant function g(x) = -15.

Solution:

$$g(x) = -15$$
$$g'(x) = 0$$

Example 2: Find the derivative of $f(t) = \pi\sqrt{3}$

Solution:

$$f(t) = \pi\sqrt{3}$$
$$f'(t) = 0$$

Note it does not matter what form the constant is in, the derivative will still be zero.

The next derivative rule that we will look at is what is called the Power Rule. The Power Rule will be used to find the derivative of functions in the form of $f(x) = x^n$, where n is a real number. The derivative of a Power function is found by multiplying the function by the exponent of n and then subtracting 1 from the exponent.

Power Rule

If $f(x) = x^n$, where n is a real number then the derivative is

$$f'(x) = \frac{d}{dx}\left(x^n\right) = n \cdot x^{n-1}$$

The Power Rule can also be used to find the derivative of simple rational expressions like $\frac{1}{x^4}$ by rewriting the expression using negative exponents as x^{-4}.

Example 3: Find the derivative of $g(t) = t^4$.

Solution:

$$g(t) = t^4$$
$$g'(t) = 4 \cdot t^{4-1}$$
$$= 4t^3$$

Example 4: Find the derivative of $h(v) = \frac{1}{v^5}$.

Solution:

First rewrite the expression in the form of v^n

$$h(v) = \frac{1}{v^5}$$

$$= v^{-5}$$

Then use the Power Rule to find the derivative

$$h'(v) = -5 \cdot v^{-5-1}$$

$$= -5v^{-6}$$

$$= -\frac{5}{v^6}$$

In the above examples, the coefficients for the functions were all equal to 1. However, the coefficient of a Power function will not always be 1. You might have functions like $-3x^5$ or $10t^{3/2}$. In cases like these, you would use the following derivative rule.

Constant times a function

If k is a real number, then the derivative of $k \cdot f(x)$ is equal to the constant "k" times the derivative of the function f(x).

$$f'(x) = \frac{d}{dx}\left[k \cdot f(x)\right] = k \cdot \frac{d}{dx} f(x)$$

Therefore, if you are given the function $f(x) = 8x^7$ then derivative would be found by multiplying the coefficient of 8 by the derivative of the function x^7, which would be found by using the Power Rule.

$$f(x) = 8x^7$$

$$f'(x) = 8 \cdot \left[\frac{d}{dx}\left(x^7\right)\right]$$

$$= 8 \cdot \left(7x^{7-1}\right)$$

$$= 56x^6$$

Example 5: Find the derivative of $f(x) = 6x^{7/3}$.

Solution:

$$f(x) = 6x^{7/3}$$

$$f'(x) = 6 \cdot \frac{d}{dx}\left(x^{7/3}\right)$$

$$= 6 \cdot \left(\frac{7}{3}x^{7/3 - 1}\right)$$

$$= 6 \cdot \left(\frac{7}{3}x^{7/3 - 3/3}\right)$$

$$= \frac{42}{3}x^{4/3}$$

$$= 14x^{4/3}$$

The last derivative rule that we will look at during this section is one that you will use for finding the derivative of polynomial functions or functions that consist of the sum and/or difference of terms in the form of the Power function x^n. The derivative of a sum or difference of terms will be equal to the sum or difference of their derivatives. In other words, when you take the derivative of such a function you will take the derivative of each individual term and add or subtract the derivatives.

Sum or Difference Rule

If $f(x) = u(x) \pm v(x)$ then

$f'(x) = \frac{d}{dx}\left[u(x)\right] \pm \frac{d}{dx}\left[v(x)\right]$ as long as $u'(x)$ and $v'(x)$ exist.

Example 6: Find the derivative of $g(x) = 11x^2 + 4x - 2$.

Solution:

The derivative of g(x) is equal to the sum/difference of the derivatives of each term.

$$
\begin{aligned}
g(x) &= 11x^2 + 4x - 2 \\
g'(x) &= \frac{d}{dx}\left(11x^2\right) + \frac{d}{dx}(4x) - \frac{d}{dx}(2) \\
&= 11 \cdot \frac{d}{dx}\left(x^2\right) + 4 \cdot \frac{d}{dx}(x) - \frac{d}{dx}(2) \\
&= 11\left(2x^{2-1}\right) + 4\left(1x^{1-1}\right) - 0 \\
&= 22x + 4
\end{aligned}
$$