Review Exercise Set 10

Exercise 1: Find dy/dx by implicit differentiation.

$$x^3 - xy + y^3 = 1$$

Exercise 2: Find dy/dx by implicit differentiation.

$$6x^2 + 3xy + 2y^2 + 17y - 6 = 0$$

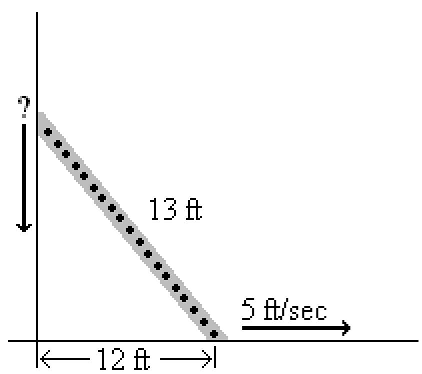
Exercise 3: Find the equation of the tangent line at the given point.

$$y^2 - 2x = 4y + 1 \; ; \; (-2, 1)$$

Exercise 4: Assume that x and y are functions of t and evaluate dy/dt.

$$x^3 - xy + y^3 = 1 \text{ ; } dx/dt = -2 \text{ ; } x = 3 \text{ ; } y = 0$$

Exercise 5: A 13-foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12 feet from the house, the base is moving at a rate of 5 feet per second. How fast is the top of the ladder sliding down the wall?



Review Exercise Set 10 Answer Key

Exercise 1: Find dy/dx by implicit differentiation.

$x^3 - xy + y^3 = 1$

$$3x^2 - \left[(x)\frac{dy}{dx} + y(1)\right] + 3y^2\frac{dy}{dx} = 0$$

$$3x^2 - x\frac{dy}{dx} - y + 3y^2\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2 + y$$

$$\frac{dy}{dx}\left(-x + 3y^2\right) = -3x^2 + y$$

$$\frac{dy}{dx} = \frac{-3x^2 + y}{-x + 3y^2}$$

$$\frac{dy}{dx} = \frac{-\left(3x^2 - y\right)}{-\left(x - 3y^2\right)}$$

$$\frac{dy}{dx} = \frac{3x^2 - y}{x - 3y^2}$$

Exercise 2: Find dy/dx by implicit differentiation.

$6x^2 + 3xy + 2y^2 + 17y - 6 = 0$

$$12x + \left[(3x)\frac{dy}{dx} + (y)(3)\right] + 4y\frac{dy}{dx} + 17\frac{dy}{dx} - 0 = 0$$

$$12x + 3x\frac{dy}{dx} + 3y + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = 0$$

$$3x\frac{dy}{dx} + 4y\frac{dy}{dx} + 17\frac{dy}{dx} = -12x - 3y$$

$$\frac{dy}{dx}\left(3x + 4y + 17\right) = -12x - 3y$$

$$\frac{dy}{dx} = \frac{-12x - 3y}{3x + 4y + 17}$$

$$\frac{dy}{dx} = -\frac{12x + 3y}{3x + 4y + 17}$$

Exercise 3: Find the equation of the tangent line at the given point.

$y^2 - 2x = 4y + 1$; (-2, 1)

Find the derivative

$$2y\frac{dy}{dx} - 2 = 4\frac{dy}{dx} + 0$$

$$2y\frac{dy}{dx} - 4\frac{dy}{dx} = 2$$

$$\frac{dy}{dx}(2y-4) = 2$$

$$\frac{dy}{dx} = \frac{2}{2y-4}$$

$$\frac{dy}{dx} = \frac{1}{y-2}$$

Find the slope of the tangent line at (-2, 1)

$$\frac{dy}{dx} = \frac{1}{y-2}$$

$$= \frac{1}{1-2}$$

$$= -1$$

Find the equation of the tangent line

$m = -1$ and $(x_1, y_1) = (-2, 1)$

$y - y_1 = m(x - x_1)$
$y - 1 = -1(x - (-2))$
$y - 1 = -x - 2$
$y = -x - 1$

Exercise 4: Assume that x and y are functions of t and evaluate dy/dt.

$x^3 - xy + y^3 = 1$; $dx/dt = -2$; $x = 3$; $y = 0$

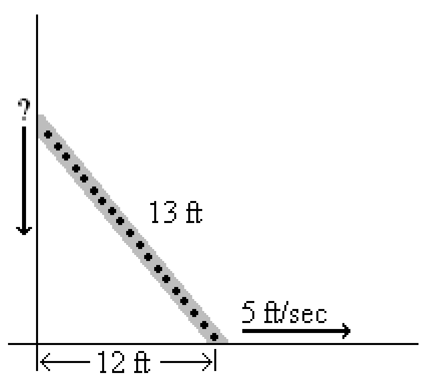
Take the implicit derivative

$$3x^2\frac{dx}{dt} - \left[(x)\frac{dy}{dt} - (y)\frac{dx}{dt}\right] + 3y^2\frac{dy}{dt} = 0$$
$$3x^2\frac{dx}{dt} - x\frac{dy}{dt} + y\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$$

Substitute in the given values and solve for dy/dt

$$3x^2\frac{dx}{dt} - x\frac{dy}{dt} + y\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0$$
$$3(3)^2(-2) - (3)\frac{dy}{dt} + (0)(-2) + 3(0)^2\frac{dy}{dt} = 0$$
$$-54 - 3\frac{dy}{dt} = 0$$
$$-3\frac{dy}{dt} = 54$$
$$\frac{dy}{dt} = -18$$

Exercise 5: A 13-foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12 feet from the house, the base is moving at a rate of 5 feet per second. How fast is the top of the ladder sliding down the wall?



Exercise 5 (Continued):

An equation can be setup based on the relationship between the length of the ladder, the height the top of ladder is above the ground, and the distance the bottom of the ladder is from the wall. These three distances form a right triangle so we can use the Pythagorean Theorem.

$c^2 = a^2 + b^2$ where

c = length of the ladder = 13 ft
a = distance from the wall = 12 ft
b = height above the ground = unknown

Since the length of the ladder is constant we will substitute this value into the equation before taking the implicit derivative.

$13^2 = a^2 + b^2$
$169 = a^2 + b^2$

Next we need to find the value of b when a = 12

$169 = a^2 + b^2$
$169 = 12^2 + b^2$
$169 = 144 + b^2$
$25 = b^2$
$5 = b$

Take implicit derivative

$$0 = 2a\frac{da}{dt} + 2b\frac{db}{dt}$$

Now substitute in the known values

$$0 = 2a\frac{da}{dt} + 2b\frac{db}{dt}$$

$$0 = 2(12)(5) + 2(5)\frac{db}{dt}$$

$$0 = 120 + 10\frac{db}{dt}$$

$$-120 = 10\frac{db}{dt}$$

$$-12 = \frac{db}{dt}$$

The top of the ladder is sliding down the wall at a rate of 12 ft/sec.