Review Exercise Set 6

Exercise 1: Find the derivative of the given function.

$$f(x) = (3x^2 + 2)(2x - 1)$$

Exercise 2: Find the derivative of the given function.

$$f(x) = (3t^3 - 5t)(\sqrt[3]{t^2} - 1)$$

Exercise 3: Find the derivative of the given function.

$$f(x) = \frac{x^2 - 4x + 2}{x + 3}$$

Exercise 4: Find the derivative of the given function.

$$f(x) = \frac{\sqrt{x}}{4x^2 + x - 3}$$

Exercise 5: Find the derivative of the given function.

$$f(x) = \frac{(3x^2 - 2)(2x + 5)}{5x - 3}$$

Review Exercise Set 6 Answer Key

Exercise 1: Find the derivative of the given function.

$$f(x) = (3x^2 + 2)(2x - 1)$$

Find the derivative of each term

$$D_x(3x^2 + 2) = 6x + 0 = 6x$$

$$D_x(2x - 1) = 2 - 0 = 2$$

Apply the product rule and substitute in the derivatives of the terms

$$f'(x) = (3x^2 + 2)D_x(2x - 1) + (2x - 1)D_x(3x^2 + 2)$$
$$f'(x) = (3x^2 + 2)(2) + (2x - 1)(6x)$$

Simplify the derivative

$$f'(x) = 6x^2 + 4 + 12x^2 - 6x$$
$$f'(x) = 18x^2 - 6x + 4$$

Exercise 2: Find the derivative of the given function.

$$f(x) = (3t^3 - 5t)(\sqrt[3]{t^2} - 1)$$

Find the derivative of each term

$$D_x(3t^3 - 5t) = 9t^2 - 5$$

$$D_x(\sqrt[3]{t^2} - 1) = D_x(t^{2/3} - 1) = \frac{2}{3} t^{-1/3}$$

Apply the product rule and substitute in the derivatives of the terms

$$f'(x) = (3t^3 - 5t)D_x(\sqrt[3]{t^2} - 1) + (\sqrt[3]{t^2} - 1)D_x(3t^3 - 5t)$$
$$f'(x) = (3t^3 - 5t)(\frac{2}{3} t^{-1/3}) + (t^{2/3} - 1)(9t^2 - 5)$$

Simplify the derivative

$$f'(x) = 2t^{8/3} - \frac{10}{3}t^{2/3} + 9t^{8/3} - 5t^{2/3} - 9t^2 + 5$$
$$f'(x) = 11t^{8/3} - \frac{25}{3} t^{2/3} - 9t^2 + 5$$

Exercise 3: Find the derivative of the given function.

$$f(x) = \frac{x^2 - 4x + 2}{x + 3}$$

Find the derivative of the numerator and denominator

$D_x(x^2 - 4x + 2) = 2x - 4$

$D_x(x + 3) = 1$

Apply the quotient rule and substitute in the derivatives of the terms

$$f'(x) = \frac{(x+3)D_x(x^2 - 4x + 2) - (x^2 - 4x + 2)D_x(x+3)}{(x+3)^2}$$

$$= \frac{(x+3)(2x-4) - (x^2 - 4x + 2)(1)}{(x+3)^2}$$

Simplify the derivative

$$f'(x) = \frac{2x^2 + 2x - 12 - x^2 + 4x - 2}{(x+3)^2}$$

$$= \frac{x^2 + 6x - 14}{(x+3)^2}$$

Exercise 4: Find the derivative of the given function.

$$f(x) = \frac{\sqrt{x}}{4x^2 + x - 3}$$

Find the derivative of the numerator and denominator

$D_x(\sqrt{x}\ ) = D_x(x^{1/2}) = \frac{1}{2}\ x^{-1/2}$

$D_x(4x^2 + x - 3) = 8x + 1$

Exercise 4 (Continued):

Apply the quotient rule and substitute in the derivatives of the terms

$$f'(x) = \frac{\left(4x^2 + x - 3\right) D_x \sqrt{x} - \sqrt{x} D_x \left(4x^2 + x - 3\right)}{\left(4x^2 + x - 3\right)^2}$$

$$= \frac{\left(4x^2 + x - 3\right)\left(\frac{1}{2} x^{-1/2}\right) - x^{1/2}\left(8x + 1\right)}{\left(4x^2 + x - 3\right)^2}$$

Simplify the derivative

$$f'(x) = \frac{2x^{3/2} + \frac{1}{2} x^{1/2} - \frac{3}{2} x^{-1/2} - 8x^{3/2} - x^{1/2}}{\left(4x^2 + x - 3\right)^2}$$

$$= \frac{-6x^{3/2} - \frac{1}{2} x^{1/2} - \frac{3}{2} x^{-1/2}}{\left(4x^2 + x - 3\right)^2}$$

$$= \frac{-6x^{3/2} - \frac{1}{2} x^{1/2} - \frac{3}{2} x^{-1/2}}{\left(4x^2 + x - 3\right)^2} \times \frac{2x}{2x}$$

Note: multiplying the numerator and denominator by 2x with eliminate the fractions in the numerator and ensure that a radical is not present in the denominator.

$$f'(x) = \frac{-12x^{5/2} - x^{3/2} - 3x^{1/2}}{2x\left(4x^2 + x - 3\right)^2}$$

$$= -\frac{12x^{5/2} + x^{3/2} + 3x^{1/2}}{2x\left(4x^2 + x - 3\right)^2}$$

Exercise 5: Find the derivative of the given function.

$$f(x) = \frac{(3x^2 - 2)(2x + 5)}{5x - 3}$$

Find the derivative of the numerator and denominator

$D_x(3x^2 - 2)(2x + 5)$
$= (3x^2 - 2)D_x(2x + 5) + (2x + 5)D_x(3x^2 - 2)$
$= (3x^2 - 2)(2) + (2x + 5)(6x)$
$= 6x^2 - 4 + 12x^2 + 30x$
$= 18x^2 + 30x - 4$

$D_x(5x - 3) = 5$

Apply the quotient rule and substitute in the derivatives of the terms

$$f(x) = \frac{(5x-3)D_x\left[(3x^2-2)(2x+5)\right] - (3x^2-2)(2x+5)D_x(5x-3)}{(5x-3)^2}$$
$$= \frac{(5x-3)(18x^2+30x-4) - (3x^2-2)(2x+5)(5)}{(5x-3)^2}$$

Simplify the derivative

$$f(x) = \frac{90x^3 + 96x^2 - 110x + 12 - (30x^3 + 75x^2 - 20x - 50)}{(5x-3)^2}$$
$$= \frac{60x^3 + 21x^2 - 90x + 62}{(5x-3)^2}$$