Review Exercise Set 6

Exercise 1: Find the derivative of the given function.

\[ f(x) = (3x^2 + 2)(2x - 1) \]

Exercise 2: Find the derivative of the given function.

\[ f(x) = (3t^3 - 5t)(\sqrt[3]{t^2} - 1) \]

Exercise 3: Find the derivative of the given function.

\[ f(x) = \frac{x^2 - 4x + 2}{x + 3} \]
Exercise 4: Find the derivative of the given function.

\[ f(x) = \frac{\sqrt{x}}{4x^2 + x - 3} \]

Exercise 5: Find the derivative of the given function.

\[ f(x) = \frac{(3x^2 - 2)(2x + 5)}{5x - 3} \]
Exercise 1: Find the derivative of the given function.

\[ f(x) = (3x^2 + 2)(2x - 1) \]

Find the derivative of each term

\[ D_x(3x^2 + 2) = 6x + 0 = 6x \]
\[ D_x(2x - 1) = 2 - 0 = 2 \]

Apply the product rule and substitute in the derivatives of the terms

\[ f'(x) = (3x^2 + 2)D_x(2x - 1) + (2x - 1)D_x(3x^2 + 2) \]
\[ f'(x) = (3x^2 + 2)(2) + (2x - 1)(6x) \]

Simplify the derivative

\[ f'(x) = 6x^2 + 4 + 12x^2 - 6x \]
\[ f'(x) = 18x^2 - 6x + 4 \]

Exercise 2: Find the derivative of the given function.

\[ f(x) = (3t^3 - 5t)(\sqrt[3]{t^2} - 1) \]

Find the derivative of each term

\[ D_x(3t^3 - 5t) = 9t^2 - 5 \]
\[ D_x(\sqrt[3]{t^2} - 1) = D_x(t^{2/3} - 1) = \frac{2}{3} t^{-1/3} \]

Apply the product rule and substitute in the derivatives of the terms

\[ f'(x) = (3t^3 - 5t)D_x(\sqrt[3]{t^2} - 1) + (\sqrt[3]{t^2} - 1)D_x(3t^3 - 5t) \]
\[ f'(x) = (3t^3 - 5t)(\frac{2}{3} t^{-1/3}) + (t^{2/3} - 1)(9t^2 - 5) \]

Simplify the derivative

\[ f'(x) = 2t^{8/3} - \frac{10}{3} t^{2/3} + 9t^{8/3} - 5t^{2/3} - 9t^2 + 5 \]
\[ f'(x) = 11t^{8/3} - \frac{25}{3} t^{2/3} - 9t^2 + 5 \]
Exercise 3: Find the derivative of the given function.

\[ f(x) = \frac{x^2 - 4x + 2}{x + 3} \]

Find the derivative of the numerator and denominator

\[ D_x(x^2 - 4x + 2) = 2x - 4 \]
\[ D_x(x + 3) = 1 \]

Apply the quotient rule and substitute in the derivatives of the terms

\[ f''(x) = \frac{(x + 3)D_x(x^2 - 4x + 2) - (x^2 - 4x + 2)D_x(x + 3)}{(x + 3)^2} \]
\[ = \frac{(x + 3)(2x - 4) - (x^2 - 4x + 2)(1)}{(x + 3)^2} \]

Simplify the derivative

\[ f''(x) = \frac{2x^2 + 2x - 12 - x^2 + 4x - 2}{(x + 3)^2} \]
\[ = \frac{x^2 + 6x - 14}{(x + 3)^2} \]

Exercise 4: Find the derivative of the given function.

\[ f(x) = \frac{\sqrt{x}}{4x^2 + x - 3} \]

Find the derivative of the numerator and denominator

\[ D_x(\sqrt{x}) = D_x(x^{1/2}) = \frac{1}{2}x^{1/2} \]
\[ D_x(4x^2 + x - 3) = 8x + 1 \]
Exercise 4 (Continued):

Apply the quotient rule and substitute in the derivatives of the terms

\[
f'(x) = \frac{(4x^2 + x - 3) D_x \sqrt{x} - \sqrt{x} D_x (4x^2 + x - 3)}{(4x^2 + x - 3)^2} = \frac{(4x^2 + x - 3) \left( \frac{1}{2} x^{-1/2} \right) - x^{1/2} (8x + 1)}{(4x^2 + x - 3)^2}
\]

Simplify the derivative

\[
f'(x) = \frac{2x^{3/2} + \frac{1}{2} x^{1/2} - 2x^{3/2} - 8x^{3/2} - x^{1/2}}{(4x^2 + x - 3)^2} = \frac{-6x^{3/2} - \frac{1}{2} x^{1/2} - \frac{3}{2} x^{-1/2}}{(4x^2 + x - 3)^2} = \frac{-6x^{3/2} - \frac{1}{2} x^{1/2} - \frac{3}{2} x^{-1/2}}{(4x^2 + x - 3)^2} \times \frac{2x}{2x}
\]

Note: multiplying the numerator and denominator by 2x with eliminate the fractions in the numerator and ensure that a radical is not present in the denominator.

\[
f'(x) = \frac{-12x^{5/2} - x^{3/2} - 3x^{1/2}}{2x (4x^2 + x - 3)^2} = \frac{-12x^{5/2} + x^{3/2} + 3x^{1/2}}{2x (4x^2 + x - 3)^2}
\]
Exercise 5: Find the derivative of the given function.

\[ f(x) = \frac{(3x^2 - 2)(2x + 5)}{5x - 3} \]

Find the derivative of the numerator and denominator

\[
D_x(3x^2 - 2)(2x + 5) = (3x^2 - 2)D_x(2x + 5) + (2x + 5)D_x(3x^2 - 2) \\
= (3x^2 - 2)(2) + (2x + 5)(6x) \\
= 6x^2 - 4 + 12x^2 + 30x \\
= 18x^2 + 30x - 4
\]

\[ D_x(5x - 3) = 5 \]

Apply the quotient rule and substitute in the derivatives of the terms

\[
f(x) = \frac{(5x - 3)\left[(3x^2 - 2)(2x + 5)\right] - (3x^2 - 2)(2x + 5)D_x(5x - 3)}{(5x - 3)^2}
\]

\[
= \frac{(5x - 3)(18x^2 + 30x - 4) - (3x^2 - 2)(2x + 5)(5)}{(5x - 3)^2}
\]

Simplify the derivative

\[
f(x) = \frac{90x^3 + 96x^2 - 110x + 12 - (30x^3 + 75x^2 - 20x - 50)}{(5x - 3)^2}
\]

\[
= \frac{60x^3 + 21x^2 - 90x + 62}{(5x - 3)^2}
\]