Review Exercise Set 8

Exercise 1: The total cost (in hundreds of dollars) to produce $x$ units of a product is defined by the given function.

\[ C(x) = \frac{3x + 2}{x + 4} \]

a) Find the marginal cost function
b) What is the marginal cost when $x = 20$

Exercise 2: The profit (in hundreds of dollars) from the sale of $x$ units of a product is defined by the given function. Find and interpret the marginal profit when 30 units are sold.

\[ P(x) = \frac{x^2}{x - 1}; \text{ where } x > 1 \]

Exercise 3: Suppose the cost in dollars of producing $x$ units is given by the function $C(x) = .2x^2 + 6x + 50$. Find the average cost function and the average marginal cost function.
Exercise 4: Suppose the cost in dollars of producing $x$ units is given by the function $C(x) = (x^3 + 3)^3$. Find the average marginal cost function and evaluate it when $x = 10$.

Exercise 5: A company has the given cost and revenue functions where $x$ is between 0 and 1000. Find the marginal profit function and evaluate it when $x = 500$

\[ C(x) = 1000x - .2x^2; \quad R(x) = .0008x^3 - 2.4x^2 + 2400x \]
Review Exercise Set 8 Answer Key

Exercise 1: The total cost (in hundreds of dollars) to produce x units of a product is defined by the given function.

\[ C(x) = \frac{3x + 2}{x + 4} \]

a) Find the marginal cost function

\[ C'(x) = \frac{(x + 4)D_x(3x + 2) - (3x + 2)D_x(x + 4)}{(x + 4)^2} \]

\[ = \frac{(x + 4)(3) - (3x + 2)(1)}{(x + 4)^2} \]

\[ = \frac{3x + 12 - 3x - 2}{(x + 4)^2} \]

\[ = \frac{10}{(x + 4)^2} \]

b) What is the marginal cost when x = 20

\[ C'(x) = \frac{10}{(x + 4)^2} \]

\[ C'(20) = \frac{10}{(20 + 4)^2} \]

\[ = \frac{10}{576} \]

\[ = \frac{5}{288} \]
Exercise 2: The profit (in hundreds of dollars) from the sale of $x$ units of a product is defined by the given function. Find and interpret the marginal profit when 30 units are sold.

$$P(x) = \frac{x^2}{x-1}; \text{ where } x > 1$$

Find the derivative

$$P'(x) = \frac{(x-1)D_x(x^2) - (x^2)D_x(x-1)}{(x-1)^2}$$

$$= \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2}$$

$$= \frac{2x^2 - 2x - x^2}{(x-1)^2}$$

$$= \frac{x^2 - 2x}{(x-1)^2}$$

Evaluate the marginal profit function at $x = 30$

$$P'(30) = \frac{30^2 - 2(30)}{(30-1)^2}$$

$$= \frac{900 - 60}{841}$$

$$= \frac{840}{841}$$

$$\approx 0.99881$$

Interpret results

The profit from selling the 31st unit is approximately $99.88.
Exercise 3: Suppose the cost in dollars of producing \( x \) units is given by the function \( C(x) = .2x^2 + 6x + 50 \). Find the average cost function and the marginal average cost function.

Find the average cost function

\[
\overline{C}(x) = \frac{C(x)}{x} = \frac{.2x^2 + 6x + 50}{x} = .2x + 6 + \frac{50}{x}
\]

Find the marginal average cost function

\[
\overline{C}'(x) = .2 + 0 - 50x^{-2} = .2 - \frac{50}{x^2}
\]

Exercise 4: Suppose the cost in dollars of producing \( x \) units is given by the function \( C(x) = (x^2 + 3)^3 \). Find the marginal average cost function and evaluate it when \( x = 10 \).

Find the average cost function

\[
\overline{C}(x) = \frac{C(x)}{x} = \frac{(x^2 + 3)^3}{x}
\]

Find the marginal average cost function

\[
\overline{C}'(x) = \frac{(x)D_x \left( (x^2 + 3)^3 \right) - (x^2 + 3)^3 D_x \left( x \right)}{(x)^2}
\]
\[
= \frac{(x)(3)(x^2 + 3)^2(2x) - (x^2 + 3)^3(1)}{x^2}
\]
\[
= \frac{6x^2(x^2 + 3)^2 - (x^2 + 3)^3}{x^2}
\]
\[
= \frac{(x^2 + 3)^2 \left[ 6x^2 - (x^2 + 3) \right]}{x^2}
\]
\[
= \frac{(x^2 + 3)^2(5x^2 - 3)}{x^2}
\]
Exercise 4 (Continued):

Evaluate the marginal average cost function at \( x = 10 \)

\[
C''(x) = \frac{(x^2 + 3)^2 \left( 5x^2 - 3 \right)}{x^2}
\]

\[
C''(10) = \frac{(10^2 + 3)^2 \left( 5(10)^2 - 3 \right)}{10^2}
\]

\[
= \frac{(103)^2 (497)}{100}
\]

\[
= 52,726.73
\]

Exercise 5: A company has the given cost and revenue functions where \( x \) is between 0 and 1000.

Find the marginal profit function and evaluate it when \( x = 500 \)

\( C(x) = 1000x - 0.2x^2; R(x) = 0.0008x^3 - 2.4x^2 + 2400x \)

Find the profit function

\[
P(x) = R(x) - C(x)
\]

\[
= (.0008x^3 - 2.4x^2 + 2400x) - (1000x - 0.2x^2)
\]

\[
= .0008x^3 - 2.2x^2 + 1400x
\]

Find the marginal profit function

\[
P'(x) = .0008x^3 - 2.2x^2 + 1400x
\]

\[
= .0024x^2 - 4.4x + 1400
\]

Evaluate the marginal profit function at \( x = 500 \)

\[
P'(500) = .0024(500)^2 - 4.4(500) + 1400
\]

\[
= 600 - 2200 + 1400
\]

\[
= -200
\]