

Review Exercise Set 9

Exercise 1: Find the second derivative of the given function.

$$f(x) = -x^4 + 2x^3 - x^2$$

Exercise 2: Find the second derivative of the given function.

$$g(x) = \frac{x^2}{x-1}$$

Exercise 3: Find the third derivative of the given function.

$$h(x) = \frac{3x-1}{x+2}$$

Exercise 4: Suppose a ball is shot directly upward with a velocity of 256 ft/sec. Its height above the ground after t seconds is given by the function $s(t) = 256t - 16t^2$. Find the velocity and acceleration functions.

Exercise 5: Maria forgets to set the parking brake on her car and it begins to roll down a hill. The distance the car travels (in feet) from its starting point is given by the function $s(t) = 1.5t^2 + 4t$, where t is in seconds.

- a) What is the velocity of the car at 4 seconds?
- b) What is the acceleration at 4 seconds?

Review Exercise Set 9 Answer Key

Exercise 1: Find the second derivative of the given function.

$$f(x) = -x^4 + 2x^3 - x^2$$

$$f'(x) = -4x^3 + 6x^2 - 2x$$

$$f''(x) = -12x^2 + 12x - 2$$

Exercise 2: Find the second derivative of the given function.

$$g(x) = \frac{x^2}{x-1}$$

First derivative

$$\begin{aligned} g'(x) &= \frac{(x-1)D_x(x^2) - (x^2)D_x(x-1)}{(x-1)^2} \\ &= \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2} \\ &= \frac{2x^2 - 2x - x^2}{(x-1)^2} \\ &= \frac{x^2 - 2x}{(x-1)^2} \end{aligned}$$

Second derivative

$$\begin{aligned} g''(x) &= \frac{(x-1)^2 D_x(x^2 - 2x) - (x^2 - 2x) D_x(x-1)^2}{[(x-1)^2]^2} \\ &= \frac{(x-1)^2(2x-2) - (x^2-2x)2(x-1)(1)}{(x-1)^4} \\ &= \frac{(x-1)^2(2)(x-1) - 2(x^2-2x)(x-1)}{(x-1)^4} \\ &= \frac{2(x-1)^3 - 2(x^2-2x)(x-1)}{(x-1)^4} \\ &= \frac{2(x-1)[(x-1)^2 - (x^2-2x)]}{(x-1)^4} \end{aligned}$$

Exercise 2 (Continued):

$$\begin{aligned}g''(x) &= \frac{2[x^2 - 2x + 1 - x^2 + 2x]}{(x-1)^3} \\ &= \frac{2(1)}{(x-1)^3} \\ &= \frac{2}{(x-1)^3}\end{aligned}$$

Exercise 3: Find the third derivative of the given function.

$$h(x) = \frac{3x-1}{x+2}$$

First derivative

$$\begin{aligned}h'(x) &= \frac{(x+2)D_x(3x-1) - (3x-1)D_x(x+2)}{(x+2)^2} \\ &= \frac{(x+2)(3) - (3x-1)(1)}{(x+2)^2} \\ &= \frac{3x+6-3x+1}{(x+2)^2} \\ &= \frac{7}{(x+2)^2}\end{aligned}$$

Second derivative

$$\begin{aligned}h'(x) &= 7(x+2)^{-2} \\ h''(x) &= -14(x+2)^{-3}(1) \\ &= -\frac{14}{(x+2)^3}\end{aligned}$$

Third derivative

$$\begin{aligned}h''(x) &= -14(x+2)^{-3} \\ h'''(x) &= 42(x+2)^{-4}(1) \\ &= \frac{42}{(x+2)^4}\end{aligned}$$

Exercise 4: Suppose a ball is shot directly upward with a velocity of 256 ft/sec. Its height above the ground after t seconds is given by the function $s(t) = 256t - 16t^2$. Find the velocity and acceleration functions.

Find the velocity (first derivative of the distance function)

$$s(t) = 256t - 16t^2$$

$$s'(t) = 256 - 32t$$

$$v(t) = s'(t)$$

$$v(t) = 256 - 32t$$

Find the acceleration (second derivative of the distance function)

$$s'(t) = 256 - 32t$$

$$s''(t) = -32$$

$$a(t) = s''(t)$$

$$a(t) = -32$$

Exercise 5: Maria forgets to set the parking brake on her car and it begins to roll down a hill. The distance the car travels (in feet) from its starting point is given by the function $s(t) = 1.5t^2 + 4t$, where t is in seconds.

a) What is the velocity of the car at 4 seconds?

Find $v(t) = s'(t)$

$$s(t) = 1.5t^2 + 4t$$

$$s'(t) = 3t + 4$$

$$v(t) = 3t + 4$$

Evaluate $v(t)$ when $t = 4$

$$v(4) = 3(4) + 4$$

$$v(4) = 16$$

At $t = 4$ seconds, the velocity of the car is 16 ft/sec.

b) What is the acceleration at 4 seconds?

Find $a(t) = s''(t) = v'(t)$

$$v(t) = 3t + 4$$

$$v'(t) = 3$$

$$a(t) = 3$$

Exercise 5 (Continued):

Evaluate $a(t)$ when $t = 4$

$$a(4) = 3$$

$$a(4) = 3$$

At $t = 4$ seconds, the acceleration of the car is 3 ft/sec^2 .