Review Exercise Set 9

Exercise 1: Find the second derivative of the given function.

\[ f(x) = -x^4 + 2x^3 - x^2 \]

Exercise 2: Find the second derivative of the given function.

\[ g(x) = \frac{x^2}{x-1} \]

Exercise 3: Find the third derivative of the given function.

\[ h(x) = \frac{3x-1}{x+2} \]
Exercise 4: Suppose a ball is shot directly upward with a velocity of 256 ft/sec. Its height above the ground after $t$ seconds is given by the function $s(t) = 256t - 16t^2$. Find the velocity and acceleration functions.

Exercise 5: Maria forgets to set the parking brake on her car and it begins to roll down a hill. The distance the car travels (in feet) from its starting point is given by the function $s(t) = 1.5t^2 + 4t$, where $t$ is in seconds.

a) What is the velocity of the car at 4 seconds?
b) What is the acceleration at 4 seconds?
Review Exercise Set 9 Answer Key

Exercise 1: Find the second derivative of the given function.

\[ f(x) = -x^4 + 2x^3 - x^2 \]

\[ f'(x) = -4x^3 + 6x^2 - 2x \]

\[ f''(x) = -12x^2 + 12x - 2 \]

Exercise 2: Find the second derivative of the given function.

\[ g(x) = \frac{x^2}{x-1} \]

First derivative

\[ g'(x) = \frac{(x-1)D_x(x^2) - (x^2)D_x(x-1)}{(x-1)^2} \]

\[ = \frac{(x-1)(2x) - (x^2)(1)}{(x-1)^2} \]

\[ = \frac{2x^2 - 2x - x^2}{(x-1)^2} \]

\[ = \frac{x^2 - 2x}{(x-1)^2} \]

Second derivative

\[ g''(x) = \frac{(x-1)^2 D_x(x^2 - 2x) - (x^2 - 2x)D_x(x-1)^2}{[(x-1)^2]^2} \]

\[ = \frac{(x-1)^2(2x-2) - (x^2 - 2x)(2(x-1)(1))}{(x-1)^4} \]

\[ = \frac{(x-1)^2(2)(x-1) - 2(x^2 - 2x)(x-1)}{(x-1)^4} \]

\[ = \frac{2(x-1)^3 - 2(x^2 - 2x)(x-1)}{(x-1)^4} \]

\[ = \frac{2(x-1)[(x-1)^2 - (x^2 - 2x)]}{(x-1)^4} \]
Exercise 2 (Continued):

\[ g''(x) = \frac{2\left[ x^2 - 2x + 1 - x^2 + 2x \right]}{(x-1)^3} \]

\[ = \frac{2(1)}{(x-1)^3} \]

\[ = \frac{2}{(x-1)^3} \]

Exercise 3: Find the third derivative of the given function.

\[ h(x) = \frac{3x-1}{x+2} \]

First derivative

\[ h'(x) = \frac{(x+2)D_x(3x-1)-(3x-1)D_x(x+2)}{(x+2)^2} \]

\[ = \frac{(x+2)(3)-(3x-1)(1)}{(x+2)^2} \]

\[ = \frac{3x+6-3x+1}{(x+2)^2} \]

\[ = \frac{7}{(x+2)^2} \]

Second derivative

\[ h''(x) = 7(x+2)^{-2} \]

\[ h'''(x) = -14(x+2)^{-3} (1) \]

\[ = -\frac{14}{(x+2)^3} \]

Third derivative

\[ h''(x) = -14(x+2)^{-3} \]

\[ h'''(x) = 42(x+2)^{-4} (1) \]

\[ = \frac{42}{(x+2)^4} \]
Exercise 4: Suppose a ball is shot directly upward with a velocity of 256 ft/sec. Its height above the ground after t seconds is given by the function \( s(t) = 256t - 16t^2 \). Find the velocity and acceleration functions.

Find the velocity (first derivative of the distance function)

\[
s(t) = 256t - 16t^2
\]
\[
s'(t) = 256 - 32t
\]
\[
v(t) = s'(t)
\]
\[
v(t) = 256 - 32t
\]

Find the acceleration (second derivative of the distance function)

\[
s'(t) = 256 - 32t
\]
\[
s''(t) = -32
\]
\[
a(t) = s''(t)
\]
\[
a(t) = -32
\]

Exercise 5: Maria forgets to set the parking brake on her car and it begins to roll down a hill. The distance the car travels (in feet) from its starting point is given by the function \( s(t) = 1.5t^2 + 4t \), where \( t \) is in seconds.

a) What is the velocity of the car at 4 seconds?

Find \( v(t) = s'(t) \)

\[
s(t) = 1.5t^2 + 4t
\]
\[
s'(t) = 3t + 4
\]
\[
v(t) = 3t + 4
\]

Evaluate \( v(t) \) when \( t = 4 \)

\[
v(4) = 3(4) + 4
\]
\[
v(4) = 16
\]

At \( t = 4 \) seconds, the velocity of the car is 16 ft/sec.

b) What is the acceleration at 4 seconds?

Find \( a(t) = s''(t) = v'(t) \)

\[
v(t) = 3t + 4
\]
\[
v'(t) = 3
\]
\[
a(t) = 3
\]
Exercise 5 (Continued):

Evaluate $a(t)$ when $t = 4$

\[ a(4) = 3 \]
\[ a(4) = 3 \]

At $t = 4$ seconds, the acceleration of the car is $3 \text{ ft/sec}^2$. 