Review Exercise Set 17

Exercise 1: Find the derivative of the given exponential function.

$f(x) = e^{-2x+1}$

Exercise 2: Find the derivative of the given exponential function.

$h(x) = e^{2x^3 - x}$

Exercise 3: Find the derivative of the given exponential function.

$s(x) = (8x^2 - 4x)^3 / (e^x + e^{-x})$

Exercise 4: The cost (in hundreds of dollars) to produce x units can be approximated with the cost function

$$C(x) = [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{1/2}$$

Find the marginal cost when the production level is at 10 units.

Exercise 5: Since 1950, the growth in world population (in millions) closely fits the exponential function defined by

$$P(t) = 2600\ e^{.018t}$$

where t is the number of years since 1950. Find the instantaneous rate of change in the world population in the year 2010.

Review Exercise Set 17 Answer Key

Exercise 1: Find the derivative of the given exponential function.

$f(x) = e^{-2x+1}$

$f'(x) = e^{-2x+1} * D_x(-2x + 1)$
$f'(x) = e^{-2x+1} * (-2)$
$f'(x) = -2e^{-2x+1}$

Exercise 2: Find the derivative of the given exponential function.

$$h(x) = e^{2x^3-x}$$
$$h'(x) = e^{2x^3-x} D_x\left(2x^3 - x\right)$$
$$= e^{2x^3-x}\left(6x^2 - 1\right)$$

Exercise 3: Find the derivative of the given exponential function.

$s(x) = (8x^2 - 4x)^3 / (e^x + e^{-x})$

Find derivative of numerator and denominator

$D_x(8x^2 - 4x)^3 = 3 * (8x^2 - 4x)^2 * (16x - 4)$
$D_x(8x^2 - 4x)^3 = (48x - 12)(8x^2 - 4x)^2$

$D_x(e^x + e^{-x}) = e^x D_x(x) + e^{-x} D_x(-x)$
$D_x(e^x + e^{-x}) = e^x(1) + e^{-x}(-1)$
$D_x(e^x + e^{-x}) = e^x - e^{-x}$

Apply the quotient rule to s(x)

$$s'(x) = \frac{\left(e^x + e^{-x}\right) D_x\left(8x^2 - 4x\right)^3 - \left(8x^2 - 4x\right)^3 D_x\left(e^x + e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}$$
$$= \frac{\left(e^x + e^{-x}\right)\left(48x - 12\right)\left(8x^2 - 4x\right)^2 - \left(8x^2 - 4x\right)^3\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}$$
$$= \frac{\left(8x^2 - 4x\right)^2\left[\left(e^x + e^{-x}\right)\left(48x - 12\right) - \left(8x^2 - 4x\right)\left(e^x - e^{-x}\right)\right]}{\left(e^x + e^{-x}\right)^2}$$
$$= \frac{\left(8x^2 - 4x\right)^2\left[52xe^x - 12e^x + 44xe^{-x} - 12e^{-x} - 8x^2e^x + 8x^2e^{-x}\right]}{\left(e^x + e^{-x}\right)^2}$$

Exercise 4: The cost (in hundreds of dollars) to produce x units can be approximated with the cost function

$$C(x) = [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{1/2}$$

Find the marginal cost when the production level is at 10 units.

Find the derivative of C(x)

$$C'(x) = 1/2 * [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{-1/2} * Dx(800 - 200(1.6)^{-.1x + .9})$$
$$C'(x) = 1/2 * [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{-1/2} * [-200(1.6)^{-.1x + .9} * Dx(-.1x+.9)]$$
$$C'(x) = 1/2 * [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{-1/2} * [-200(1.6)^{-.1x + .9} * (-.1)]$$
$$C'(x) = 1/2 * [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{-1/2} * (20(1.6)^{-.1x + .9})$$
$$C'(x) = (10(1.6)^{-.1x + .9})\ [\ 800 - 200(1.6)^{-.1x + .9}\ ]^{-1/2}$$

Evaluate the derivative at x = 10

$$C'(10) = (10(1.6)^{-.1(10) + .9})\ [\ 800 - 200(1.6)^{-.1(10) + .9}\ ]^{-1/2}$$
$$C'(10) = (10(1.6)^{-1 + .9})\ [\ 800 - 200(1.6)^{-1 + .9}\ ]^{-1/2}$$
$$C'(10) = (10(1.6)^{-.1})\ [\ 800 - 200(1.6)^{-.1}\ ]^{-1/2}$$
$$C'(10) \approx (9.54)\ [800 - 190.82]^{-1/2}$$
$$C'(10) \approx (9.54)\ [609.18]^{-1/2}$$
$$C'(10) \approx 0.3865$$

The marginal cost at a production level of 10 units is $38.65.

Exercise 5: Since 1950, the growth in world population (in millions) closely fits the exponential function defined by

$$P(t) = 2600\ e^{.018t}$$

where t is the number of years since 1950. Find the instantaneous rate of change in the world population in the year 2010.

Find the derivative of P(t)

$$P'(t) = 2600\ e^{.018t} * D_x(0.018t)$$
$$P'(t) = 2600\ e^{.018t} * (0.018)$$
$$P'(t) = 46.8\ e^{.018t}$$

Evaluate the derivative

t = number of years since 1950
t = 2010 - 1950
t = 60

$$P'(60) = 46.8\ e^{.018(60)}$$
$$P'(60) = 46.8\ e^{1.08}$$
$$P'(60) \approx 137.81$$