Review Exercise Set 17

Exercise 1: Find the derivative of the given exponential function.

\[ f(x) = e^{-2x+1} \]

Exercise 2: Find the derivative of the given exponential function.

\[ h(x) = e^{2x^3-x} \]

Exercise 3: Find the derivative of the given exponential function.

\[ s(x) = (8x^2 - 4x)^3 / (e^x + e^{-x}) \]
Exercise 4: The cost (in hundreds of dollars) to produce \( x \) units can be approximated with the cost function

\[
C(x) = \left[ 800 - 200(1.6)^{-1x + 6} \right]^{1/2}
\]

Find the marginal cost when the production level is at 10 units.

Exercise 5: Since 1950, the growth in world population (in millions) closely fits the exponential function defined by

\[
P(t) = 2600 e^{0.018t}
\]

where \( t \) is the number of years since 1950. Find the instantaneous rate of change in the world population in the year 2010.
Review Exercise Set 17 Answer Key

Exercise 1: Find the derivative of the given exponential function.

\[ f(x) = e^{-2x+1} \]

\[ f'(x) = e^{-2x+1} \cdot D_x(-2x + 1) \]

\[ f'(x) = e^{-2x+1} \cdot (-2) \]

\[ f'(x) = -2e^{-2x+1} \]

Exercise 2: Find the derivative of the given exponential function.

\[ h(x) = e^{2x^3-x} \]

\[ h'(x) = e^{2x^3-x} D_x(2x^3-x) \]

\[ = e^{2x^3-x} \left(6x^2-1\right) \]

Exercise 3: Find the derivative of the given exponential function.

\[ s(x) = (8x^2 - 4x)^3 / (e^x + e^{-x}) \]

Find derivative of numerator and denominator

\[ D_x(8x^2 - 4x)^3 = 3 \cdot (8x^2 - 4x)^2 \cdot (16x - 4) \]

\[ D_x(8x^2 - 4x)^3 = (48x - 12)(8x^2 - 4x)^2 \]

\[ D_x(e^x + e^{-x}) = e^x D_x(x) + e^{-x} D_x(-x) \]

\[ D_x(e^x + e^{-x}) = e^x(1) + e^{-x}(-1) \]

\[ D_x(e^x + e^{-x}) = e^x - e^{-x} \]

Apply the quotient rule to \( s(x) \)

\[ s'(x) = \frac{(e^x + e^{-x}) D_x(8x^2 - 4x)^3 - (8x^2 - 4x)^3 D_x(e^x + e^{-x})}{(e^x + e^{-x})^2} \]

\[ = \frac{(e^x + e^{-x})(48x-12)(8x^2 - 4x)^2 - (8x^2 - 4x)^3 (e^x - e^{-x})}{(e^x + e^{-x})^2} \]

\[ = \frac{(8x^2 - 4x)^2 \left[ (e^x + e^{-x})(48x-12) - (8x^2 - 4x)(e^x - e^{-x}) \right]}{(e^x + e^{-x})^2} \]

\[ = \frac{(8x^2 - 4x)^2 \left[ 52xe^x - 12e^x + 44xe^{-x} - 12e^{-x} - 8x^2 e^x + 8x^2 e^{-x} \right]}{(e^x + e^{-x})^2} \]
Exercise 4: The cost (in hundreds of dollars) to produce \( x \) units can be approximated with the cost function

\[
C(x) = \left[ 800 - 200(1.6)^{-1.1x + .9} \right]^{1/2}
\]

Find the marginal cost when the production level is at 10 units.

Find the derivative of \( C(x) \)

\[
C'(x) = \frac{1}{2} \cdot \left[ 800 - 200(1.6)^{-1.1x + .9} \right]^{-1/2} \cdot D_x(800 - 200(1.6)^{-1.1x + .9})
\]

\[
C'(x) = \frac{1}{2} \cdot \left[ 800 - 200(1.6)^{-1.1x + .9} \right]^{-1/2} \cdot [-200(1.6)^{-1.1x + .9} \cdot D_x(-1.1x + .9)]
\]

\[
C'(x) = \frac{1}{2} \cdot \left[ 800 - 200(1.6)^{-1.1x + .9} \right]^{-1/2} \cdot [-200(1.6)^{-1.1x + .9} \cdot (-1)]
\]

\[
C'(x) = \frac{1}{2} \cdot \left[ 800 - 200(1.6)^{-1.1x + .9} \right]^{-1/2} \cdot (20(1.6)^{-1.1x + .9})
\]

\[
C'(x) = (10(1.6)^{-1.1x + .9}) \cdot \left[ 800 - 200(1.6)^{-1.1x + .9} \right]^{1/2}
\]

Evaluate the derivative at \( x = 10 \)

\[
C'(10) = (10(1.6)^{-1.1(10) + .9}) \cdot \left[ 800 - 200(1.6)^{-1.1(10) + .9} \right]^{1/2}
\]

\[
C'(10) = \left( \frac{9.54}{609.18} \right) \cdot \left[ 800 - 190.82 \right]^{1/2}
\]

\[
C'(10) \approx 0.3865
\]

The marginal cost at a production level of 10 units is $38.65.

Exercise 5: Since 1950, the growth in world population (in millions) closely fits the exponential function defined by

\[
P(t) = 2600 \cdot e^{0.018t}
\]

where \( t \) is the number of years since 1950. Find the instantaneous rate of change in the world population in the year 2010.

Find the derivative of \( P(t) \)

\[
P'(t) = 2600 \cdot e^{0.018t} \cdot D_x(0.018t)
\]

\[
P'(t) = 2600 \cdot e^{0.018t} \cdot (0.018)
\]

\[
P'(t) \approx 46.8 \cdot e^{0.018t}
\]

Evaluate the derivative

\[
t = \text{number of years since 1950}
\]

\[
t = 2010 - 1950
\]

\[
t = 60
\]

\[
P'(60) = 46.8 \cdot e^{0.018(60)}
\]

\[
P'(60) = 46.8 \cdot e^{1.08}
\]

\[
P'(60) \approx 137.81
\]