Functions and Mathematical Models

Polynomial functions

A polynomial function is one that has the independent variable "x" raised to a nonnegative integer exponent. The degree of a polynomial function is defined by the power (exponent) of the leading term. So let us begin by looking at the general form of a polynomial function.

A polynomial function of degree n is defined by:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where a_n, a_{n-1}, … , a_1, and a_0 are real numbers called coefficients and n is a nonnegative integer. a_n is called the leading coefficient and cannot be zero.

Examples of polynomials:

Polynomial Function	Degree	Name
$F(x) = 3x - 5$	1	Linear
$F(x) = -x^2 + 3x - 2$	2	Quadratic
$F(x) = x^3 - 4x + 10$	3	Cubic

As you can see the first three degrees of polynomials have specific names associated with them. A linear function is a 1st degree polynomial, a quadratic function is a 2nd degree polynomial, and a cubic function is a 3rd degree polynomial.

Rational functions

A rational function can be viewed as the quotient of two polynomial functions p(x) and q(x) where $q(x) \neq 0$.

$$f(x) = \frac{p(x)}{q(x)}$$

Any values for x that make q(x) zero must be excluded from the domain of the rational function f(x).

Examples of rational functions:

$$f(x)=\frac{2x^2+3}{x^2-1}$$ Two quadratic functions

$$f(x)=\frac{x^4}{4x^3-2x}$$ Power function and a cubic function

$$f(x)=\frac{x^3-x^2+5x-3}{3x+2}$$ Cubic function and a linear function

Power functions

A power function is similar to a polynomial in that it has the independent variable raised to an exponent. However, the difference between them is that the exponent in a power function is not restricted to being only nonnegative integers and a power function will consist of a single term.

$$F(x) = x^r$$

Examples of power functions:

$f(x)=x^5$ positive integer exponent

$f(x)=x^{-4}$ negative integer exponent

$f(x)=x^{1/3}$ positive fractional exponent

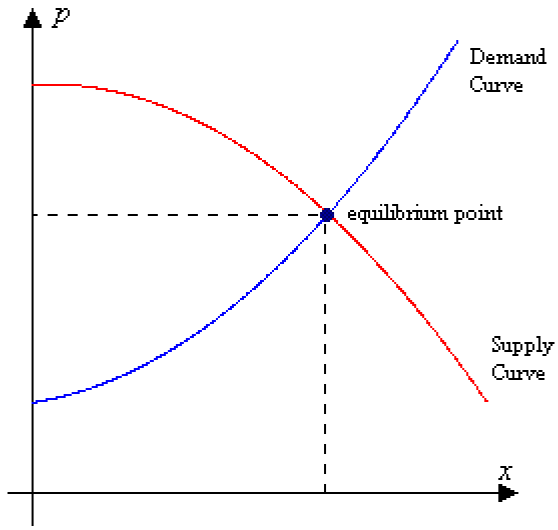
$f(x)=x^{-1/5}$ negative fractional exponent

Economic models

For supply and demand functions, the price is expressed in terms of the quantity because in economic graphs the price (or dollars) is plotted along the vertical axis. However, in reality the price is the independent variable because the quantity supplied and demanded depends on what the price.

In our supply and demand functions, both the price and quantity are restricted to being only nonnegative values. The demand function will be a downward curve since the demand for a product will be low when the price is high and will be high when the price is low. The supply curve will be just the opposite and will be an upward curve. The point where the two curves intersect is the equilibrium point.

At this point, the quantity demanded will equal the quantity supplied and the price is the equilibrium price.



Example: If a company has a demand equation of $5x + 2p - 74 = 0$ and a supply equation of $7x - 3p + 33 = 0$, where p is the unit price in dollars and x represents the quantity in thousands of units, determine the equilibrium quantity and price.

Solution:

Step 1: Solve the demand and supply equations for p

$$5x + 2p - 74 = 0$$
$$2p = -5x + 74$$
$$p = -5/2\ x + 37$$

$$7x - 3p + 33 = 0$$
$$-3p = -7x - 33$$
$$p = 7/3\ x + 11$$

Step 2: Set the two equations equal to each other and solve for x

$$-5/2\ x + 37 = 7/3\ x + 11$$
$$6(-5/2\ x + 37 = 7/3\ x + 11)$$
$$-15x + 222 = 14x + 66$$
$$156 = 29x$$
$$5.379 = x$$

Example (Continued)

Step 3: Substitute the equilibrium quantity into either equation to solve for p

$p = -5/2\ x + 37$
$p = -5/2\ (5.379) + 37$
$p = -13.4475 + 37$
$p = 23.55$

The equilibrium quantity is 5379 units with an equilibrium price of $23.55.