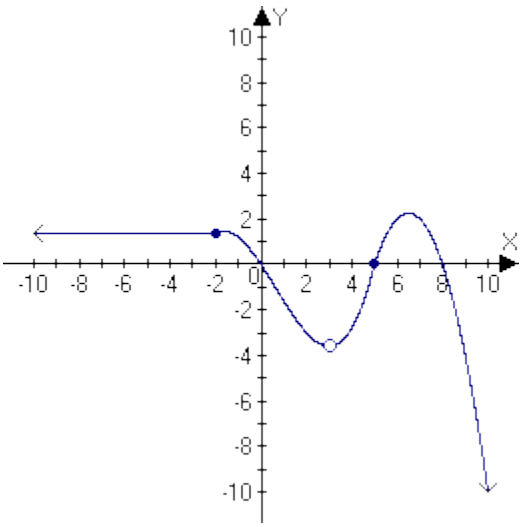
Review Exercise Set 3

Exercise 1: Find all values x = a where the function is discontinuous. For each point of discontinuity, find $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, $\lim_{x\to a} f(x)$, and f(a) if they exist.

$$f(x)=\begin{cases}1.4 & \text{where } x\le -2\\ .1x^3-.2x^2-1.5x & \text{where } -2<x<3 \text{ or } 3<x<5\\ -x^2+13x-40 & \text{where } x\ge 5\end{cases}$$



Exercise 2: Find the indicated one-sided limit, if it exists.

$$\lim_{x\to 1^+}\frac{\sqrt{4x}}{|x-1|}$$

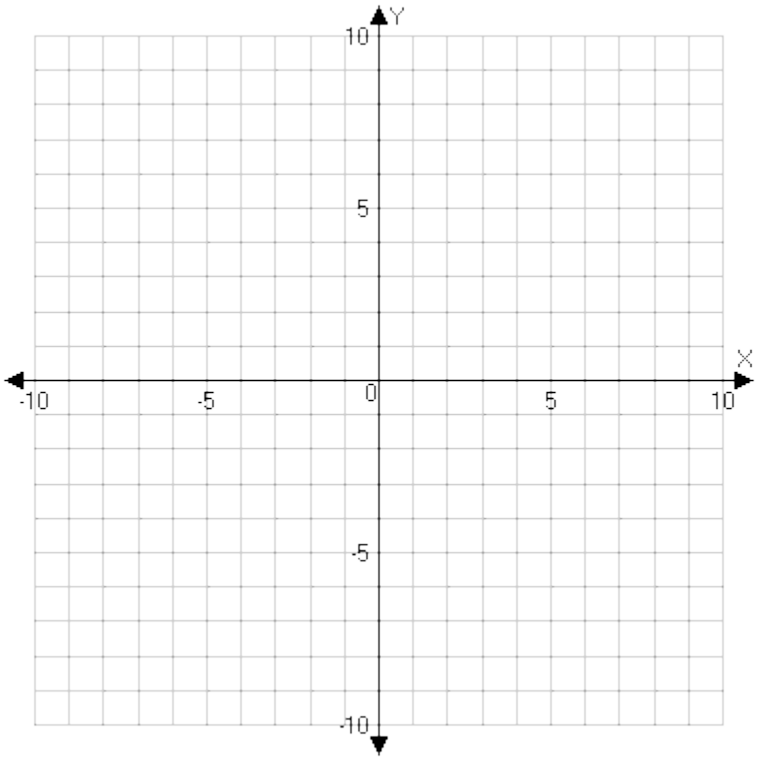
Exercise 3: Find the indicated one-sided limit, if it exists.

$$\lim_{x \to 1^+} \sqrt{\frac{x-1}{x+2}}$$

Exercise 4: Use the piecewise function defined below to:

a) sketch the graph of the function
b) find all values of x where the function is discontinuous
c) find the limits from the left and right of the values for x found in part b

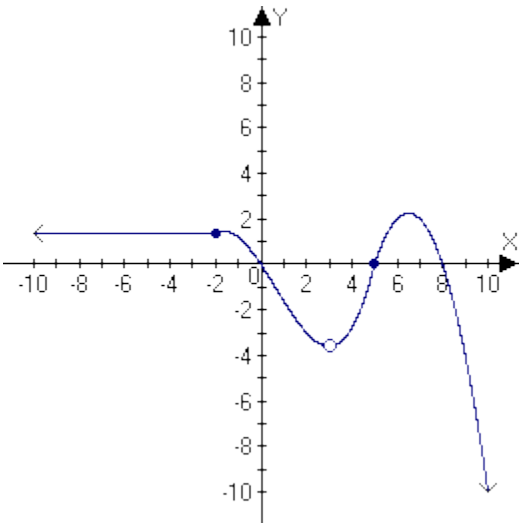
$$f(x) = \begin{cases} -2x+3 & if\ x < 1 \\ x-2 & if\ x \geq 1 \end{cases}$$



Review Exercise Set 3 Answer Key

Exercise 1: Find all values x = a where the function is discontinuous. For each point of discontinuity, find $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, $\lim_{x\to a} f(x)$, and f(a) if they exist.

$$f(x)=\begin{cases}1.4 & where\ \ x\le -2\\ .1x^3-.2x^2-1.5x & where\ \ -2<x<3\ \ or\ \ 3<x<5\\ -x^2+13x-40 & where\ \ x\ge 5\end{cases}$$



The function is discontinuous at x = 3.

$$\lim_{x\to 3^-} f(x)=-3 \qquad \lim_{x\to 3^+} f(x)=-3 \qquad \lim_{x\to 3} f(x)=-3$$

f(3) does not exist

Exercise 2: Find the indicated one-sided limit, if it exists.

$$\lim_{x\to 1^+}\frac{\sqrt{4x}}{|x-1|}$$

Evaluate the expression with a value just larger than 1

$$\approx\frac{\sqrt{4(1.0001)}}{|1.0001-1|}\approx\frac{2}{0.0001}\approx 20{,}000$$

As x gets closer to 1 the value of the expression will continue to increase without bound and approach infinity. Therefore, the one-sided limit does not exist.

Exercise 3: Find the indicated one-sided limit, if it exists.

$$\lim_{x\to 1^+}\sqrt{\frac{x-1}{x+2}}$$

Evaluate the expression with a value just larger than 1

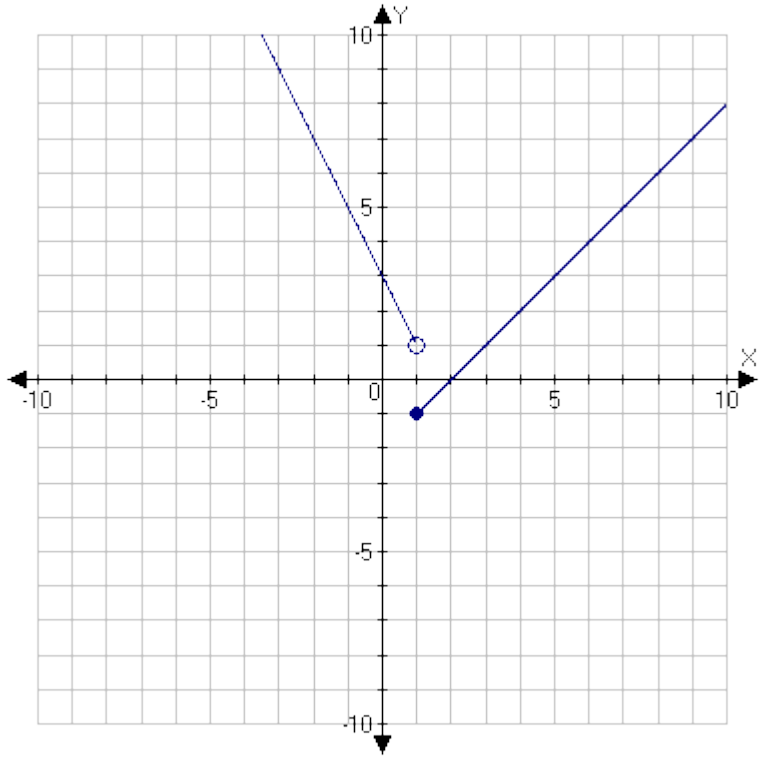
$$\approx\sqrt{\frac{1.0001-1}{1.0001+2}}\approx\sqrt{\frac{0.0001}{3.0001}}\approx\sqrt{0}\approx 0$$

The one-sided limit approaches the value of 0 as x approaches 1 from the right.

Exercise 4: Use the piecewise function defined below to:

a) sketch the graph of the function
b) find all values of x where the function is discontinuous
c) find the limits from the left and right of the values for x found in part b

$$f(x)=\begin{cases}-2x+3 & if\ x<1\\ x-2 & if\ x\geq 1\end{cases}$$



a)

b) The function would be discontinuous at x = 1

c) $\lim_{x\to 1^-} f(x)=1$ and $\lim_{x\to 1^+} f(x)=-1$