Review Exercise Set 4

Exercise 1: Find the average rate of change for the function over the given interval.

\[ f(x) = x^2 + x - 12 \] over the interval [-2, 4]

Exercise 2: Find the instantaneous rate of change for the function at the given value.

\[ f(x) = \frac{3x}{x - 4} \] at \( x = 2 \)

Exercise 3: Using the definition for the derivative, find \( f'(x) \).

\[ f(x) = 2x^2 + 3x \]
Exercise 4: Using the definition for the derivative, find \( f'(x) \) and then evaluate it at \( x = -3 \).

\[
f(x) = \frac{2}{x + 4}
\]
Review Exercise Set 4 Answer Key

Exercise 1: Find the average rate of change for the function over the given interval.

\[ f(x) = x^2 + x - 12 \] over the interval \([-2, 4]\)

\[ \text{Average rate of change} = \frac{f(x + h) - f(x)}{h} \]

Find \( h \).

\( h \) is the difference between the ending and starting interval values

\[ h = 4 - (-2) = 6 \]

Find the average rate of change

\[ \frac{f(x + h) - f(x)}{h} = \frac{f(4) - f(-2)}{6} \]

\[ = \frac{(4^2 + 4 - 12) - [(-2)^2 + (-2) - 12]}{6} \]

\[ = \frac{8 - (-10)}{6} \]

\[ = 3 \]

Exercise 2: Find the instantaneous rate of change for the function at the given value.

\[ f(x) = \frac{3x}{x - 4} \] at \( x = 2 \)

\[ \text{Instantaneous rate of change} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

Find \( f(x + h) \)

\[ f(x + h) = \frac{3(x + h)}{(x + h) - 4} \]

\[ = \frac{3x + 3h}{x + h - 4} \]
Exercise 2 (Continued):

Find \( f(x + h) - f(x) \)

\[
\begin{align*}
f(x + h) - f(x) &= \frac{3x + 3h}{x + h - 4} - \frac{3x}{x - 4} \\
&= \frac{(3x + 3h)(x - 4) - 3x(x + h - 4)}{(x + h - 4)(x - 4)} \\
&= \frac{3x^2 - 12x + 3hx - 12h - 3x^2 - 3hx + 12x}{(x + h - 4)(x - 4)} \\
&= \frac{-12h}{(x + h - 4)(x - 4)}
\end{align*}
\]

Find \( \frac{f(x + h) - f(x)}{h} \)

\[
\begin{align*}
\frac{f(x + h) - f(x)}{h} &= \frac{-12h}{(x + h - 4)(x - 4)} \times \frac{1}{h} \\
&= \frac{-12}{(x + h - 4)(x - 4)}
\end{align*}
\]

Find \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \)

\[
\begin{align*}
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} &= \lim_{h \to 0} \frac{-12}{(x + h - 4)(x - 4)} \\
&= \frac{-12}{(x + 0 - 4)(x - 4)} \\
&= \frac{-12}{(x - 4)^2}
\end{align*}
\]

Evaluate at \( x = 2 \)

\[
\frac{-12}{(x - 4)^2} = \frac{-12}{(2 - 4)^2} = \frac{-12}{4} = -3
\]

The instantaneous rate of change when \( x = 2 \) is -1.
Exercise 3: Using the definition for the derivative, find \( f'(x) \).

\[ f(x) = 2x^2 + 3x \]

Find \( f(x + h) \)

\[ f(x + h) = 2(x + h)^2 + 3(x + h) \]
\[ f(x + h) = 2(x^2 + 2xh + h^2) + 3x + 3h \]
\[ f(x + h) = 2x^2 + 4xh + 2h^2 + 3x + 3h \]

Find \( f(x + h) - f(x) \)

\[ f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 3x + 3h) - (2x^2 + 3x) \]
\[ f(x + h) - f(x) = 2x^2 + 4xh + 2h^2 + 3x + 3h - 2x^2 - 3x \]
\[ f(x + h) - f(x) = 4xh + 2h^2 + 3h \]

Find \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \)

\[ \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3 \]

Find \( f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \)

\[ f'(x) = 4x + 3 \]
Exercise 4: Using the definition for the derivative, find \( f'(x) \) and then evaluate it at \( x = -3 \).

\[
f(x) = \frac{2}{x + 4}
\]

Find \( f(x + h) \)

\[
f(x + h) = \frac{2}{x + h + 4}
\]

Find \( f(x + h) - f(x) \)

\[
f(x + h) - f(x) = \frac{2}{x + h + 4} - \frac{2}{x + 4}
= \frac{2(x + 4) - 2(x + h + 4)}{(x + h + 4)(x + 4)}
= \frac{2x + 8 - 2x - 2h - 8}{(x + h + 4)(x + 4)}
= \frac{-2h}{(x + h + 4)(x + 4)}
\]

Find \( \frac{f(x + h) - f(x)}{h} \)

\[
\frac{f(x + h) - f(x)}{h} = \frac{-2h}{(x + h + 4)(x + 4)} \times \frac{1}{h}
= \frac{-2}{(x + h + 4)(x + 4)}
\]

Find \( \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \)

\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{-2}{(x + h + 4)(x + 4)}
= \frac{-2}{(x + 0 + 4)(x + 4)}
= \frac{-2}{(x + 4)^2}
\]
Exercise 4 (Continued):

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ = \frac{-2}{(x+4)^2} \]

Evaluate the derivative \( f'(x) \) at \( x = 2 \)

\[ f'(x) = \frac{-2}{(x+4)^2} \]

\[ f'(2) = \frac{-2}{(2+4)^2} \]

\[ = \frac{-2}{36} \]

\[ = -\frac{1}{18} \]