Review Exercise Set 19

Exercise 1: Find the indefinite integral.

$$\int \left(2x^3 + 4x^2 - 5x + 7\right) dx$$

Exercise 2: Find the indefinite integral.

$$\int \left(4e^x - \frac{2}{x}\right) dx$$

Exercise 3: Find an equation of the function whose tangent line has a slope of $f'(x) = x^2 - x - 2$ and passes through the point (2, -4).

Exercise 4: Find the cost function if the marginal cost function is $3(2x)^{1/2}$ and fixed costs are $10.

Exercise 5: The marginal weekly profit is given by $-.06x^2 + 600$, where x is the number of units sold per week. If the weekly profit is $7,500 when 50 units are sold, find the weekly profit function.

Review Exercise Set 19 Answer Key

Exercise 1: Find the indefinite integral.

$$\begin{aligned}\int\left(2x^3+4x^2-5x+7\right)dx &= \int\left(2x^3\right)dx+\int\left(4x^2\right)dx-\int(5x)\,dx+\int(7)\,dx\\ &= 2\int\left(x^3\right)dx+4\int\left(x^2\right)dx-5\int(x)\,dx+\int(7)\,dx\\ &= 2\left(\frac{1}{4}x^4\right)+4\left(\frac{1}{3}x^3\right)-5\left(\frac{1}{2}x^2\right)+7x+C\\ &= \frac{1}{2}x^4+\frac{4}{3}x^3-\frac{5}{2}x^2+7x+C\end{aligned}$$

Exercise 2: Find the indefinite integral.

$$\begin{aligned}\int\left(4e^x-\frac{2}{x}\right)dx &= \int\left(4e^x\right)dx-\int\left(\frac{2}{x}\right)dx\\ &= 4\int\left(e^x\right)dx-2\int\left(\frac{1}{x}\right)dx\\ &= 4e^x-2\ln|x|+C\end{aligned}$$

Exercise 3: Find an equation of the function whose tangent line has a slope of $f'(x) = x^2 - x - 2$ and passes through the point (2, -4).

Find the general antiderivative

$$\begin{aligned}F(x) &= \int\left(x^2-x-2\right)dx\\ &= \int\left(x^2\right)dx-\int(x)\,dx-\int(2)\,dx\\ &= \frac{1}{3}x^3-\frac{1}{2}x^2-2x+C\end{aligned}$$

Determine the value of C by substituting the values of the given point into the antiderivative

$$\begin{aligned}F(x) &= \frac{1}{3}x^3-\frac{1}{2}x^2-2x+C\\ F(2) &= \frac{1}{3}(2)^3-\frac{1}{2}(2)^2-2(2)+C\\ -4 &= \frac{1}{3}(8)-\frac{1}{2}(4)-4+C\end{aligned}$$

Exercise 3 (Continued):

$$-4 = \frac{8}{3} - 2 - 4 + C$$

$$-4 - \frac{8}{3} + 2 + 4 = C$$

$$-\frac{2}{3} = C$$

Substitute the value of C into the general antiderivative to obtain the equation of the function.

$$F(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x - \frac{2}{3}$$

Exercise 4: Find the cost function if the marginal cost function is $3(2x)^{1/2}$ and fixed costs are \$10.

Find the general antiderivative

$$\begin{aligned} C(x) &= \int 3(2x)^{1/2}\,dx \\ &= 3(2)^{1/2}\int (x)^{1/2}\,dx \\ &= 3(2)^{1/2}\frac{2}{3}(x)^{3/2} + C \\ &= (2)^{1/2}(2)(x)^{3/2} + C \\ &= (2)^{3/2}(x)^{3/2} + C \\ &= (2x)^{3/2} + C \end{aligned}$$

Determine the value of C

The fixed costs are the costs when x = 0

$$\begin{aligned} C(x) &= (2x)^{3/2} + C \\ C(0) &= (2\times 0)^{3/2} + C \\ 10 &= 0 + C \\ 10 &= C \end{aligned}$$

Substitute the value of C into the general antiderivative to obtain the equation of the function.

$C(x) = (2x)^{3/2} + 10$

Exercise 5: The marginal weekly profit is given by $-.06x^2 + 600$, where x is the number of units sold per week. If the weekly profit is \$7,500 when 50 units are sold, find the weekly profit function.

Find the general antiderivative

$$\begin{aligned} P(x) &= \int \left(-0.06x^2 + 600\right) dx \\ &= \int \left(-0.06x^2\right) dx + \int (600)\, dx \\ &= -0.06 \int \left(x^2\right) dx + \int (600)\, dx \\ &= -0.06\left(\frac{1}{3}x^3\right) + 600x + C \\ &= -0.02x^3 + 600x + C \end{aligned}$$

Determine the value of C

$P(50) = -.02(50)^3 + 600(50) + C$
$7500 = -2500 + 30000 + C$
$7500 = 27500 + C$
$-20000 = C$

Substitute the value of C into the general antiderivative to obtain the equation of the function.

$P(x) = -.02x^3 + 600x - 20000$