Review Exercise Set 19

Exercise 1: Find the indefinite integral.

\[ \int \left( 2x^3 + 4x^2 - 5x + 7 \right) dx \]

Exercise 2: Find the indefinite integral.

\[ \int \left( 4e^x - \frac{2}{x} \right) dx \]

Exercise 3: Find an equation of the function whose tangent line has a slope of
\[ f'(x) = x^2 - x - 2 \] and passes through the point (2, -4).
Exercise 4: Find the cost function if the marginal cost function is $3(2x)^{1/2}$ and fixed costs are $10$.

Exercise 5: The marginal weekly profit is given by $-0.06x^2 + 600$, where $x$ is the number of units sold per week. If the weekly profit is $7,500$ when 50 units are sold, find the weekly profit function.
**Review Exercise Set 19 Answer Key**

**Exercise 1:** Find the indefinite integral.
\[
\int \left(2x^3 + 4x^2 - 5x + 7\right) dx = \int 2x^3 dx + \int 4x^2 dx - \int 5x dx + \int 7 dx
\]
\[
= 2 \int \left(\frac{1}{4} x^4\right) + 4 \int \left(\frac{1}{3} x^3\right) - 5 \int \left(\frac{1}{2} x^2\right) + 7x + C
\]
\[
= \frac{1}{2} x^4 + \frac{4}{3} x^3 - \frac{5}{2} x^2 + 7x + C
\]

**Exercise 2:** Find the indefinite integral.
\[
\int \left(4e^x - \frac{2}{x}\right) dx = \int 4e^x dx - \int \left(\frac{2}{x}\right) dx
\]
\[
= 4 \int e^x dx - 2 \int \left(\frac{1}{x}\right) dx
\]
\[
= 4e^x - 2 \ln |x| + C
\]

**Exercise 3:** Find an equation of the function whose tangent line has a slope of \(f'(x) = x^2 - x - 2\) and passes through the point \((2, -4)\).

Find the general antiderivative
\[
F(x) = \int \left(x^2 - x - 2\right) dx
\]
\[
= \int x^3 dx - \int x dx - \int 2 dx
\]
\[
= \frac{1}{3} x^3 - \frac{1}{2} x^2 - 2x + C
\]

Determine the value of \(C\) by substituting the values of the given point into the antiderivative
\[
F(x) = \frac{1}{3} x^3 - \frac{1}{2} x^2 - 2x + C
\]
\[
F(2) = \frac{1}{3} (2)^3 - \frac{1}{2} (2)^2 - 2(2) + C
\]
\[
-4 = \frac{1}{3} (8) - \frac{1}{2} (4) - 4 + C
\]
Exercise 3 (Continued):

\[-4 = \frac{8}{3} - 2 - 4 + C \]
\[-4 = \frac{8}{3} + 2 + 4 = C \]
\[-\frac{2}{3} = C \]

Substitute the value of C into the general antiderivative to obtain the equation of the function.

\[ F(x) = \frac{1}{3} x^3 - \frac{1}{2} x^2 - 2x - \frac{2}{3} \]

Exercise 4: Find the cost function if the marginal cost function is \( 3(2x)^{1/2} \) and fixed costs are $10.

Find the general antiderivative

\[ C(x) = \int 3(2x)^{1/2} \, dx \]
\[ = 3(2)^{1/2} \int (x)^{1/2} \, dx \]
\[ = 3(2)^{1/2} \, \frac{2}{3} (x)^{3/2} + C \]
\[ = (2)^{1/2} (2)(x)^{3/2} + C \]
\[ = (2)^{3/2} (x)^{3/2} + C \]
\[ = (2x)^{3/2} + C \]

Determine the value of C

The fixed costs are the costs when \( x = 0 \)

\[ C(x) = (2x)^{3/2} + C \]
\[ C(0) = (2 \times 0)^{3/2} + C \]
\[ 10 = 0 + C \]
\[ 10 = C \]

Substitute the value of C into the general antiderivative to obtain the equation of the function.

\[ C(x) = (2x)^{3/2} + 10 \]
Exercise 5: The marginal weekly profit is given by \(-0.06x^2 + 600\), where \(x\) is the number of units sold per week. If the weekly profit is $7,500 when 50 units are sold, find the weekly profit function.

Find the general antiderivative

\[
P(x) = \int (-0.06x^2 + 600) \, dx
\]
\[
= \int (-0.06x^2) \, dx + \int (600) \, dx
\]
\[
= -0.06 \int (x^2) \, dx + \int (600) \, dx
\]
\[
= -0.06 \left( \frac{1}{3} x^3 \right) + 600x + C
\]
\[
= -0.02x^3 + 600x + C
\]

Determine the value of \(C\)

\[
P(50) = -0.02(50)^3 + 600(50) + C
\]
\[
7500 = -2500 + 30000 + C
\]
\[
7500 = 27500 + C
\]
\[
-20000 = C
\]

Substitute the value of \(C\) into the general antiderivative to obtain the equation of the function.

\[
P(x) = -0.02x^3 + 600x - 20000
\]