Review Exercise Set 20

Exercise 1: Find the indefinite integral by using the substitution method.

$$\int\left(\frac{2x-9}{\sqrt{x^2-9x+1}}\right)dx$$

Exercise 2: Find the indefinite integral by using the substitution method.

$$\int\left(\frac{u}{1+9u^2}\right)du$$

Exercise 3: Find the indefinite integral by using the substitution method.

$$\int\left(3ye^{3y^2}\right)dy$$

Exercise 4: A company has determined their marginal cost function to be defined by $3x / (x^2 + e)$. If $C(10) = 7$ find the cost function.

Exercise 5: The rate of growth of the profit (in hundred thousands of dollars) is approximated by $P'(x) = 20x/(x^2 + 1)$ where x represents time measured in years. The total profit in the fifteenth year is $59.20535. Find the total profit function.

Review Exercise Set 20 Answer Key

Exercise 1: Find the indefinite integral by using the substitution method.

$$\int \left(\frac{2x-9}{\sqrt{x^2-9x+1}} \right) dx$$

Choose function to be represented by u and find its derivative

Since the radical is the more complex portion and has a higher degree power we will choose the radicand as u

$u = x^2 - 9x + 1$
$du/dx = 2x - 9$
$du = (2x - 9)\, dx$

Perform the substitution in the integral

$$\int \left(\frac{2x-9}{\sqrt{x^2-9x+1}} \right) dx = \int \frac{1}{\left(x^2-9x+1\right)^{1/2}} (2x-9)\, dx$$
$$= \int \left(x^2-9x+1\right)^{-1/2} (2x-9)\, dx$$
$$= \int u^{-1/2} du$$

Use the integration rules to find the antiderivative

$$\int \left(\frac{2x-9}{\sqrt{x^2-9x+1}} \right) dx = \frac{1}{\frac{1}{2}} u^{1/2} + C$$
$$= 2u^{1/2} + C$$

Substitute the function back in for u

$$\int \left(\frac{2x-9}{\sqrt{x^2-9x+1}} \right) dx = 2\left(x^2-9x+1\right)^{1/2} + C$$

Exercise 2: Find the indefinite integral by using the substitution method.

$$\int\left(\frac{u}{1+9u^2}\right)du$$

Choose function to be represented by t and find its derivative

Since the function already contains the variable u we will use t as our substitution variable. The denominator is the more complex portion and has a higher degree power we will choose the denominator as t.

$t = 1 + 9u^2$
dt/du = 18u
dt = 18u du
$\frac{1}{18}$ dt = u du

Perform the substitution in the integral

$$\int\left(\frac{u}{1+9u^2}\right)du = \int\frac{1}{1+9u^2}u\ du$$
$$= \int\frac{1}{t}\times\frac{1}{18}dt$$
$$= \frac{1}{18}\int\frac{1}{t}\ dt$$

Use the integration rules to find the antiderivative

$$\int\left(\frac{u}{1+9u^2}\right)du = \frac{1}{18}\ln|t| + C$$

Substitute the function back in for t

$$\int\left(\frac{u}{1+9u^2}\right)du = \frac{1}{18}\ln\left|1+9u^2\right| + C$$

Exercise 3: Find the indefinite integral by using the substitution method.

$$\int \left(3ye^{3y^2}\right) dy$$

Choose function to be represented by u and find its derivative

Let u equal the exponent for e

$u = 3y^2$
du/dy = 6y
du = 6y dy
$\frac{1}{2}$ du = 3y dy

Perform the substitution in the integral

$$\begin{aligned}\int \left(3ye^{3y^2}\right) dy &= \int e^{3y^2}(3y)\,dy \\ &= \int e^u \frac{1}{2} du \\ &= \frac{1}{2}\int e^u du\end{aligned}$$

Use the integration rules to find the antiderivative

$$\int \left(3ye^{3y^2}\right) dy = \frac{1}{2}e^u + C$$

Substitute the function back in for u

$$\int \left(3ye^{3y^2}\right) dy = \frac{1}{2}e^{3y^2} + C$$

Exercise 4: A company has determined their marginal cost function to be defined by 3x / (x^2 + e). If C(10) = 7 find the cost function.

Setup the indefinite integral to find the cost function

$$C(x) = \int \frac{3x}{x^2 + e} dx$$
$$= \int \frac{1}{x^2 + e} 3x\, dx$$

Choose function to be represented by u and find its derivative

Let the denominator of the fraction be u

u = x^2 + e
du/dx = 2x
$\frac{3}{2}$ * du = $\frac{3}{2}$ * 2x dx
$\frac{3}{2}$ du = 3x dx

Perform the substitution in the integral

$$\int \frac{1}{x^2 + e} 3x\, dx = \int \frac{1}{u} \times \frac{3}{2} du$$
$$= \frac{3}{2} \int \frac{1}{u} du$$

Use the integration rules to find the antiderivative

$$\int \frac{1}{x^2 + e} 3x\, dx = \frac{3}{2} \ln|u| + C$$

Substitute the function back in for u

$$\int \frac{1}{x^2 + e} 3x\, dx = \frac{3}{2} \ln\left|x^2 + e\right| + C$$

Determine the value of the constant C

$$C(x) = \frac{3}{2} \ln\left|x^2 + e\right| + C$$
$$C(10) = \frac{3}{2} \ln\left|(10)^2 + e\right| + C$$

Exercise 4 (Continued):

$$C(10) = \frac{3}{2}\ln\left|(10)^2 + e\right| + C$$
$$7 = \frac{3}{2}\left[\ln(10)^2 + \ln e\right] + C$$
$$7 = \frac{3}{2}\left[2\ln(10) + 1\right] + C$$
$$7 = 3\ln(10) + \frac{3}{2} + C$$
$$7 - \ln(10)^3 - \frac{3}{2} = C$$
$$\frac{14 - 2\ln(10)^3 - 3}{2} = C$$
$$\frac{11 - \ln(10)^6}{2} = C$$
$$-1.4 \approx C$$

Substitute the value of C into the general antiderivative to obtain the equation of the function.

$$C(x) = \frac{3}{2}\ln\left|x^2 + e\right| - 1.4$$

Exercise 5: The rate of growth of the profit (in hundred thousands of dollars) is approximated by P'(x) = 20x/(x^2 + 1) where x represents time measured in years. The total profit in the fifteenth year is $59.20535. Find the total profit function.

Setup the indefinite integral to find the profit function

$$P(x) = \int \frac{20x}{x^2 + 1}\,dx$$
$$= \int \frac{1}{x^2 + 1}\,20x\,dx$$

Choose function to be represented by u and find its derivative

Let the denominator of the fraction be u

u = x^2 + 1
du/dx = 2x
du = 2x dx
10 * du = 10 * 2x dx
10 du = 20x dx

Exercise 5 (Continued):

Perform the substitution in the integral

$$P(x) = \int \frac{1}{x^2+1} 20x\, dx$$
$$= \int \frac{1}{u} 10\, du$$
$$= 10\int \frac{1}{u}\, du$$

Use the integration rules to find the antiderivative

$$P(x) = 10\ln|u| + C$$

Substitute the function back in for u

$$P(x) = 10\ln\left|x^2+1\right| + C$$

Determine the value of the constant C

P(x) = 10 ln $|x^2 + 1|$ + C
P(15) = 10 ln $|(15)^2 + 1|$ + C
59.20535 = 10 ln $|(15)^2 + 1|$ + C
59.20535 = 10 ln (226) + C
59.20535 = 54.20535 + C
5 = C

Substitute the value of C into the general antiderivative to obtain the equation of the function.

P(x) = 10 ln $|x^2 + 1|$ + 5