Review Exercise Set 22

Exercise 1: Using the Fundamental Theorem of Calculus, evaluate the following integral.

$$\int_{-2}^{2}\left(x^3 - 2x + 4\right)dx$$

Exercise 2: Using the Fundamental Theorem of Calculus, evaluate the following integral.

$$\int_{0}^{\ln 2}\left(e^{3x}\right)dx$$

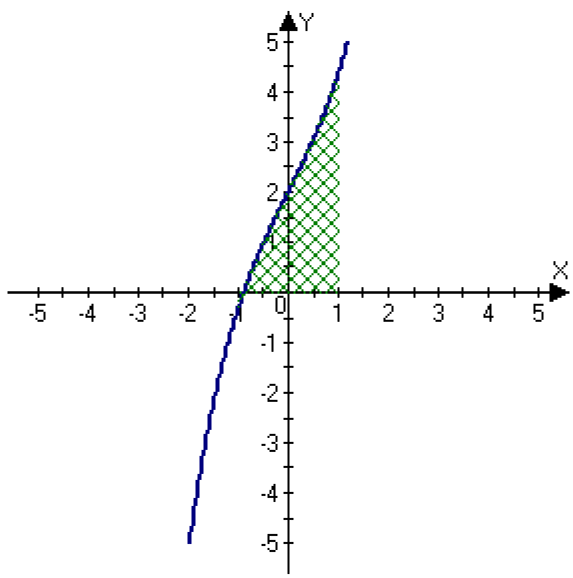
Exercise 3: Using the Fundamental Theorem of Calculus, evaluate the following integral.

$$\int_{0}^{1} xe^{x^2}\,dx$$

Exercise 4: Find the total area of the region between the given function and the x-axis over the closed interval.

$$y = x^{1/3} - x \; ; \; [-1, 8]$$

Exercise 5: Find the total area of the shaded region in the graph.



$y = e^x - e^{-x}$

Review Exercise Set 22 Answer Key

Exercise 1: Using the Fundamental Theorem of Calculus, evaluate the following integral.

$$\int_{-2}^{2} \left(x^3 - 2x + 4\right) dx = \left(\frac{1}{4}x^4 - x^2 + 4x\right)\Bigg|_{-2}^{2}$$

$$= \left[\frac{1}{4}(2)^4 - (2)^2 + 4(2)\right] - \left[\frac{1}{4}(-2)^4 - (-2)^2 + 4(-2)\right]$$

$$= (4 - 4 + 8) - (4 - 4 - 8)$$

$$= 8 + 8$$

$$= 16$$

Exercise 2: Using the Fundamental Theorem of Calculus, evaluate the following integral.

$$\int_{0}^{\ln 2} \left(e^{3x}\right) dx = \left(\frac{1}{3}e^{3x}\right)\Bigg|_{0}^{\ln 2}$$

$$= \frac{1}{3}e^{3(\ln 2)} - \frac{1}{3}e^{3(0)}$$

$$= \frac{1}{3}e^{\ln 8} - \frac{1}{3}e^{0}$$

$$= \frac{1}{3}(8) - \frac{1}{3}$$

$$= \frac{7}{3}$$

Exercise 3: Using the Fundamental Theorem of Calculus, evaluate the following integral.

$$\int_{0}^{1} xe^{x^2} dx = \int_{0}^{1} e^{x^2} x dx$$

Substitution

Let u = x^2
du/dx = 2x
du = 2x dx
$\frac{1}{2}$ du = x dx

Exercise 3 (Continued):

Define integration limits in terms of u

If x = 1: If x = 0

$u = 1^2$ $u = 0^2$
u = 1 u = 0

In this case the limits remain the same.

Perform substitution and evaluate the integral

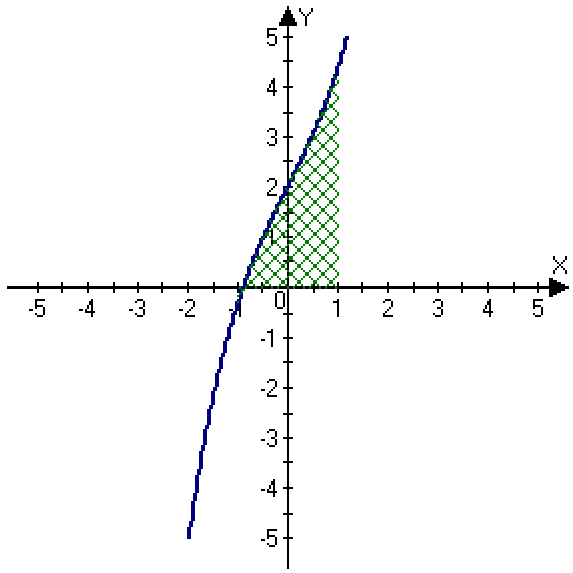
$$\int_0^1 e^{x^2} x\,dx = \int_0^1 e^u \frac{1}{2} du$$
$$= \frac{1}{2}\int_0^1 e^u du$$
$$= \frac{1}{2}\left(e^u\right)\Big|_0^1$$
$$= \frac{1}{2}\left(e^1 - e^0\right)$$
$$= \frac{1}{2}\left(e - 1\right)$$

Exercise 4: Find the total area of the region between the given function and the x-axis over the closed interval.

$y = -x^{1/3} + x$; [1, 8]

$$\int_1^8 \left(-x^{1/3} + x\right) dx = \left[-\frac{3}{4}x^{4/3} + \frac{1}{2}x^2\right]\Big|_1^8$$
$$= \left[-\frac{3}{4}(8)^{4/3} + \frac{1}{2}(8)^2\right] - \left[-\frac{3}{4}(1)^{4/3} + \frac{1}{2}(1)^2\right]$$
$$= \left[-\frac{3}{4}(16) + \frac{1}{2}(64)\right] - \left[-\frac{3}{4}(1) + \frac{1}{2}(1)\right]$$
$$= -12 + 32 + \frac{3}{4} - \frac{1}{2}$$
$$= 20\frac{1}{4}$$

Exercise 5: Find the total area of the shaded region in the graph.



$y = e^x - e^{-x} + 2$

$$\int_{-1}^{1}\left(e^{x}-e^{-x}+2\right)dx=\int_{-1}^{1}\left(e^{x}\right)dx-\int_{-1}^{1}\left(e^{-x}\right)dx+\int_{-1}^{1}\left(2\right)dx$$

$$=\left[e^{x}-\left(-e^{-x}\right)+2x\right]\Big|_{-1}^{1}$$

$$=\left[e^{x}+e^{-x}+2x\right]\Big|_{-1}^{1}$$

$$=\left[e^{(1)}+e^{-(1)}+2(1)\right]-\left[e^{(-1)}+e^{-(-1)}+2(-1)\right]$$

$$=e+e^{-1}+2-e^{-1}-e+2$$

$$=4$$