Review Exercise Set 23

Exercise 1: Evaluate the following definite integral using the properties of definite integrals.

$$\int_{1}^{3}\left(x^2-3x+2\right)dx$$

Exercise 2: Evaluate the following definite integral using the properties of definite integrals.

$$\int_{1}^{2}\left(e^{5u}-u^{-2}\right)du$$

Exercise 3: Evaluate the following definite integral using the properties of definite integrals.

$$\int_{e}^{e^2}\frac{1}{x}dx$$

Exercise 4: For the following function, use the properties of definite integrals to find the area of the region between the given function and the x-axis over the closed interval.

$f(x) = 2x^2 - 2$; [-2, 4]

Exercise 5: For the following function, use the properties of definite integrals to find the area of the region between the given function and the x-axis over the closed interval.

$f(x) = 1 - e^{-x}$; [-1, 2]

Review Exercise Set 23 Answer Key

Exercise 1: Evaluate the following definite integral using the properties of definite integrals.

$$\int_1^3 \left(x^2 - 3x + 2\right) dx = \int_1^3 \left(x^2\right) dx - \int_1^3 (3x)\, dx + \int_1^3 (2)\, dx$$

$$= \int_1^3 \left(x^2\right) dx - 3\int_1^3 (x)\, dx + 2\int_1^3 dx$$

$$= \left(\frac{1}{3}x^3\right)\Big|_1^3 - 3\left(\frac{1}{2}x^2\right)\Big|_1^3 + 2(x)\Big|_1^3$$

$$= \frac{1}{3}\left(x^3\right)\Big|_1^3 - \frac{3}{2}\left(x^2\right)\Big|_1^3 + 2(x)\Big|_1^3$$

$$= \frac{1}{3}\left(3^3 - 1^3\right) - \frac{3}{2}\left(3^2 - 1^2\right) + 2(3-1)$$

$$= \frac{1}{3}(26) - \frac{3}{2}(8) + 2(2)$$

$$= \frac{26}{3} - 12 + 4$$

$$= \frac{2}{3}$$

Exercise 2: Evaluate the following definite integral using the properties of definite integrals.

$$\int_1^2 \left(e^{5u} - u^{-2}\right) du = \int_1^2 \left(e^{5u}\right) du - \int_1^2 \left(u^{-2}\right) du$$

D_u (5u) = 5

du = $\frac{1}{5}$ * 5 du

$$\int_1^2 \left(e^{5u} - u^{-2}\right) du = \int_1^2 \left(e^{5u}\right) \frac{1}{5} \times 5du - \int_1^2 \left(u^{-2}\right) du$$

$$= \frac{1}{5}\int_1^2 \left(e^{5u}\right) 5du - \int_1^2 \left(u^{-2}\right) du$$

$$= \frac{1}{5}\left(e^{5u}\right)\Big|_1^2 - \left(-u^{-1}\right)\Big|_1^2$$

Exercise 2 (Continued):

$$\int_1^2 \left(e^{5u} - u^{-2}\right) du = \frac{1}{5}\left(e^{5u}\right)\Big|_1^2 + \left(\frac{1}{u}\right)\Bigg|_1^2$$
$$= \frac{1}{5}\left(e^{5(2)} - e^{5(1)}\right) + \left(\frac{1}{2} - 1\right)$$
$$= \frac{1}{5}e^{10} - \frac{1}{5}e^{5} - \frac{1}{2}$$

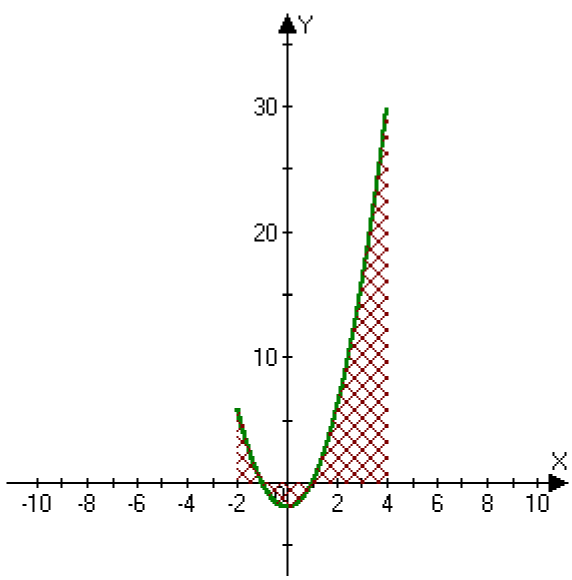
Exercise 3: Evaluate the following definite integral using the properties of definite integrals.

$$\int_e^{e^2} \frac{1}{x}\,dx = \ln|x|\Big|_e^{e^2}$$
$$= \ln\left|e^2\right| - \ln|e|$$
$$= 2 - 1$$
$$= 1$$

Exercise 4: For the following function, use the properties of definite integrals to find the area of the region between the given function and the x-axis over the closed interval.

$f(x) = 2x^2 - 2$; [-2, 4]

Sketch graph of the function on the closed interval



The area on the interval [-1, 1] is below the x-axis so we would need to split the definite integral and multiply the region on interval [-1, 1] by -1 to ensure it comes out as a positive area.

Exercise 4 (Continued):

Setup of definite integral

$$\int_{-2}^{4}\left(2x^2-2\right)dx=\int_{-2}^{-1}\left(2x^2-2\right)dx+(-1)\int_{-1}^{1}\left(2x^2-2\right)dx+\int_{1}^{4}\left(2x^2-2\right)dx$$

$$=2\int_{-2}^{-1}\left(x^2\right)dx-\int_{-2}^{-1}(2)\,dx-2\int_{-1}^{1}\left(x^2\right)dx+\int_{-1}^{1}(2)\,dx+2\int_{1}^{4}\left(x^2\right)dx-\int_{1}^{4}(2)\,dx$$

$$=\frac{2}{3}\left(x^3\right)\Big|_{-2}^{-1}-2(x)\Big|_{-2}^{-1}-\frac{2}{3}\left(x^3\right)\Big|_{-1}^{1}+2(x)\Big|_{-1}^{1}+\frac{2}{3}\left(x^3\right)\Big|_{1}^{4}-2(x)\Big|_{1}^{4}$$

$$=\frac{2}{3}\left[(-1)^3-(-2)^3\right]-2\left[-1-(-2)\right]-\frac{2}{3}\left[(1)^3-(-1)^3\right]+2\left[1-(-1)\right]$$

$$+\frac{2}{3}\left[(4)^3-(1)^3\right]-2\left[4-1\right]$$

$$=\frac{2}{3}(7)-2(1)-\frac{2}{3}(2)+2(2)+\frac{2}{3}(63)-2(3)$$

$$=\frac{2}{3}(7-2+63)-2+4-6$$

$$=\frac{2}{3}(68)-4$$

$$=\frac{136}{3}-\frac{12}{3}$$

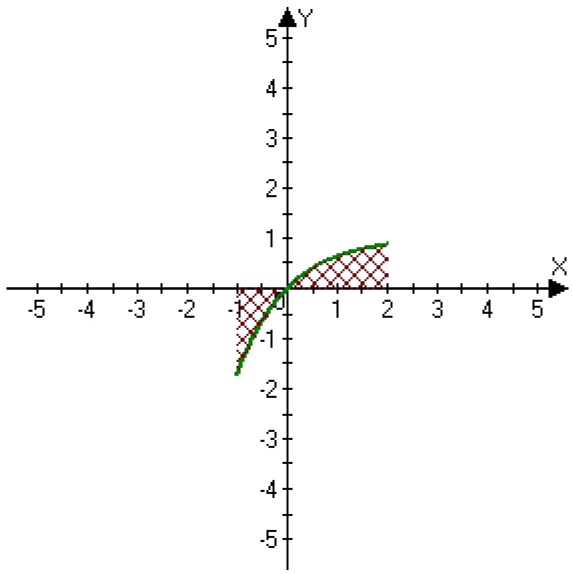
$$=\frac{124}{3}$$

$$=41\frac{1}{3}$$

Exercise 5: For the following function, use the properties of definite integrals to find the area of the region between the given function and the x-axis over the closed interval.

$f(x) = 1 - e^{-x}$; [-1, 2]

Sketch graph of the function on the closed interval



The area on the interval [-1, 0] is below the x-axis so we would need to split the definite integral and multiply the region on interval [-1, 0] by -1 to ensure it comes out as a positive area.

Setup of definite integral

$$\int_{-1}^{2}\left(1-e^{-x}\right)dx=\left(-1\right)\int_{-1}^{0}\left(1-e^{-x}\right)dx+\int_{0}^{2}\left(1-e^{-x}\right)dx$$

$$=-\int_{-1}^{0}\left(1\right)dx+\int_{-1}^{0}\left(e^{-x}\right)dx+\int_{0}^{2}\left(1\right)dx-\int_{0}^{2}\left(e^{-x}\right)dx$$

$$=-\left(x\right)\Big|_{-1}^{0}+\left(-e^{-x}\right)\Big|_{-1}^{0}+\left(x\right)\Big|_{0}^{2}-\left(-e^{-x}\right)\Big|_{0}^{2}$$

$$=-\left(x\right)\Big|_{-1}^{0}-\left(e^{-x}\right)\Big|_{-1}^{0}+\left(x\right)\Big|_{0}^{2}+\left(e^{-x}\right)\Big|_{0}^{2}$$

$$=-\left[0-\left(-1\right)\right]-\left[e^{-(0)}-e^{-(-1)}\right]+\left[2-0\right]+\left[e^{-(2)}-e^{-(0)}\right]$$

$$=-\left(1\right)-\left(1-e\right)+2+\left(e^{-2}-1\right)$$

$$=-1-1+e+2+e^{-2}-1$$

$$=e+e^{-2}-1$$

$$\approx 1.8536$$