Review Exercise Set 18

Exercise 1: Write the complex number $2-\sqrt{5}i$ in polar form.

Exercise 2: Write the complex number 6(cos 70° + i sin 70°) in rectangular form.

Exercise 3: Find the product of the given complex numbers.

z_1 = 5(cos 60° + i sin 60°)
z_2 = 3(cos 25° + i sin 25°)

Exercise 4: Find the quotient of the given complex numbers.

$z_1 = 4(\cos 56° + i \sin 56°)$
$z_2 = 7(\cos 32° + i \sin 32°)$

Exercise 5: Raise the given complex number to the third power using DeMoivre's Theorem.

$4(\cos 15° + i \sin 15°)$

Exercise 6: Find the complex square roots of the given complex number.

$16(\cos 50° + i \sin 50°)$

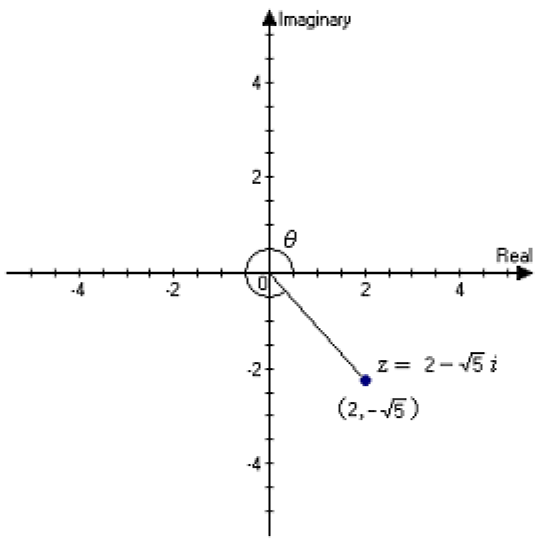
Review Exercise Set 18 Answer Key

Exercise 1: Write the complex number $2-\sqrt{5}\,i$ in polar form.

Find r

$$\begin{aligned} r &= \sqrt{a^2+b^2} \\ &= \sqrt{(2)^2+\left(-\sqrt{5}\right)^2} \\ &= \sqrt{4+5} \\ &= 3 \end{aligned}$$

Plot the complex number to determine the quadrant in which it lies



The complex number is in quadrant IV

Find θ

$$\begin{aligned} \tan\theta &= \frac{b}{a} \\ &= -\frac{\sqrt{5}}{2} \end{aligned}$$

$$\tan 48° \approx \frac{\sqrt{5}}{2}$$

Since θ is in quadrant IV, we would subtract the reference angle of 48° from 360°.

Exercise 1 (Continued):

$$\theta \approx 360° - 48°$$
$$\approx 312°$$

The polar form of $2-\sqrt{5}\,i$ is (3, 312°)

Exercise 2: Write the complex number 6(cos 70° + i sin 70°) in rectangular form. Round answer to nearest tenth.

Evaluate cos and sin at the value of theta

cos 70° = 0.3420

sin 70° = 0.9397

Substitute in the exact values of cos and sin to find the rectangular form

6(cos 70° + *i* sin 70°)
6[0.3420 + *i* (0.9397)]
6[0.3420 + *i* (0.9397)]
2.1 + 5.6*i*

z = 2.1 + 5.6*i*

Exercise 3: Find the product of the given complex numbers.

z_1 = 5(cos 60° + *i* sin 60°)
z_2 = 3(cos 25° + *i* sin 25°)

$z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$
$z_1z_2 = (5)(3)[\cos(60° + 25°) + i\sin(60° + 25°)]$
$z_1z_2 = 15(\cos 85° + i\sin 85°)$

Exercise 4: Find the quotient of the given complex numbers.

$z_1 = 4(\cos 56° + i \sin 56°)$
$z_2 = 7(\cos 32° + i \sin 32°)$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$
$$= \frac{4}{7}\left[\cos(56° - 32°) + i\sin(56° - 32°)\right]$$
$$= \frac{4}{7}(\cos 24° + i\sin 24°)$$

Exercise 5: Raise the given complex number to the third power using DeMoivre's Theorem.

$4(\cos 15° + i \sin 15°)$

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$
$$z^3 = (4)^3(\cos 3(15°) + i\sin 3(15°))$$
$$= 64(\cos 45° + i\sin 45°)$$
$$= 64\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$
$$= 32\sqrt{2} + 32\sqrt{2}\,i$$

Exercise 6: Find the complex square roots of the given complex number.

$16(\cos 50° + i \sin 50°)$

Find the first complex square root

k = 0 and n = 2

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 360°k}{n}\right) + i\sin\left(\frac{\theta + 360°k}{n}\right)\right]$$
$$z_0 = \sqrt[2]{16}\left[\cos\left(\frac{50° + 360°(0)}{2}\right) + i\sin\left(\frac{50° + 360°(0)}{2}\right)\right]$$
$$= 4(\cos 25° + i\sin 25°)$$

Exercise 6 (Continued):

Find the second complex square root

k = 1 and n = 2

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 360°k}{n}\right) + i\sin\left(\frac{\theta + 360°k}{n}\right)\right]$$

$$z_1 = \sqrt[2]{16}\left[\cos\left(\frac{50° + 360°(1)}{2}\right) + i\sin\left(\frac{50° + 360°(1)}{2}\right)\right]$$

$$= 4\left(\cos 205° + i\sin 205°\right)$$