Review Exercise Set 20

Exercise 1: Find the dot product $v \cdot w$, if $v = 2i + 10j$ and $w = 3i + 7j$.

Exercise 2: Find the dot product $3u \cdot (v - 2w)$, if $u = -7i + 6j$, $v = 5i + 3j$, and $w = -2i - 4j$.

Exercise 3: Determine if the vectors $v = -6i - 3j$ and $w = -7i + 12j$ are orthogonal.

Exercise 4: Let $v = 8i + 5j$ and $w = -2i + 4j$. Decompose v into its vector components v_1 and v_2, where v_1 is parallel to w and v_2 is orthogonal to w.

Exercise 5: A farmer is plowing a field with a mule pulling a plow. How much work is the mule doing pulling the plow the length of a field that is 275 feet long, if it is pulling the plow with a constant force of 240 lb and the straps make an angle of 28° to with the horizontal?

Review Exercise Set 20 Answer Key

Exercise 1: Find the dot product $\boldsymbol{v} \cdot \boldsymbol{w}$, if $\boldsymbol{v} = 2\boldsymbol{i} + 10\boldsymbol{j}$ and $\boldsymbol{w} = 3\boldsymbol{i} + 7\boldsymbol{j}$.

$\boldsymbol{v} \cdot \boldsymbol{w} = a_1a_2 + b_1b_2$

$\boldsymbol{v} \cdot \boldsymbol{w} = (2)(3) + (10)(7)$
$\boldsymbol{v} \cdot \boldsymbol{w} = 6 + 70$
$\boldsymbol{v} \cdot \boldsymbol{w} = 76$

Exercise 2: Find the dot product $3\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{w})$, if $\boldsymbol{u} = -7\boldsymbol{i} + 6\boldsymbol{j}$, $\boldsymbol{v} = 5\boldsymbol{i} + 3\boldsymbol{j}$, and $\boldsymbol{w} = -2\boldsymbol{i} - 4\boldsymbol{j}$.

Find $3\boldsymbol{u}$

$3\mathbf{u} = 3(-7\boldsymbol{i} + 6\boldsymbol{j})$
$3\mathbf{u} = 3(-7\boldsymbol{i}) + 3(6\boldsymbol{j})$
$3\mathbf{u} = -21\boldsymbol{i} + 18\boldsymbol{j}$

Find $2\boldsymbol{w}$

$2\mathbf{w} = 2(-2\boldsymbol{i} - 4\boldsymbol{j})$
$2\mathbf{w} = 2(-2\boldsymbol{i}) + 2(-4\boldsymbol{j})$
$2\mathbf{w} = -4\boldsymbol{i} - 8\boldsymbol{j}$

Find $\boldsymbol{v} - 2\boldsymbol{w}$

$\boldsymbol{v} - 2\mathbf{w} = (5\boldsymbol{i} + 3\boldsymbol{j}) - (-4\boldsymbol{i} - 8\boldsymbol{j})$
$\boldsymbol{v} - 2\mathbf{w} = (5\boldsymbol{i} + 3\boldsymbol{j}) + (4\boldsymbol{i} + 8\boldsymbol{j})$
$\boldsymbol{v} - 2\mathbf{w} = (5 + 4)\boldsymbol{i} + (3 + 8)\boldsymbol{j}$
$\boldsymbol{v} - 2\mathbf{w} = 9\boldsymbol{i} + 11\boldsymbol{j}$

Find $3\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{w})$

$3\mathbf{u} = -21\boldsymbol{i} + 18\boldsymbol{j}$ and $\boldsymbol{v} - 2\mathbf{w} = 9\boldsymbol{i} + 11\boldsymbol{j}$

$3\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{w}) = a_1a_2 + b_1b_2$

$3\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{w}) = (-21)(9) + (18)(11)$
$3\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{w}) = -189 + 198$
$3\boldsymbol{u} \cdot (\boldsymbol{v} - 2\boldsymbol{w}) = 9$

Exercise 3: Determine if the vectors $\boldsymbol{v} = -6\boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{w} = -7\boldsymbol{i} + 12\boldsymbol{j}$ are orthogonal.

$\boldsymbol{v} \cdot \boldsymbol{w} = a_1a_2 + b_1b_2$

$\boldsymbol{v} \cdot \boldsymbol{w} = (-6)(-7) + (-3)(12)$
$\boldsymbol{v} \cdot \boldsymbol{w} = 42 - 36$
$\boldsymbol{v} \cdot \boldsymbol{w} = 6$

Since the dot product does not equal zero, these vectors are not orthogonal.

Exercise 4: Let $\boldsymbol{v} = 8\boldsymbol{i} + 5\boldsymbol{j}$ and $\boldsymbol{w} = -2\boldsymbol{i} + 4\boldsymbol{j}$. Decompose $\boldsymbol{v}$ into its vector components $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$, where $\boldsymbol{v}_1$ is parallel to $\boldsymbol{w}$ and $\boldsymbol{v}_2$ is orthogonal to $\boldsymbol{w}$. Use the dot product of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ to check your answer.

Find the magnitude of $\boldsymbol{w}$

$$\begin{aligned}\|w\| &= \sqrt{a^2 + b^2} \\ &= \sqrt{(-2)^2 + (4)^2} \\ &= \sqrt{4+16} \\ &= \sqrt{20}\end{aligned}$$

Find the dot product of $\boldsymbol{v}$ and $\boldsymbol{w}$

$\boldsymbol{v} \cdot \boldsymbol{w} = a_1a_2 + b_1b_2$

$\boldsymbol{v} \cdot \boldsymbol{w} = (8)(-2) + (5)(4)$
$\boldsymbol{v} \cdot \boldsymbol{w} = -16 + 20$
$\boldsymbol{v} \cdot \boldsymbol{w} = 4$

Find $\boldsymbol{v}_1$

$$\begin{aligned}v_1 &= \frac{v \cdot w}{\|w\|^2} w \\ &= \frac{4}{\left\|\sqrt{20}\right\|^2}(-2i + 4j) \\ &= \frac{4}{20}(-2i + 4j) \\ &= \frac{1}{5}(-2i + 4j) \\ &= -\frac{2}{5}i + \frac{4}{5}j\end{aligned}$$

Exercise 4 (Continued):

Find $\boldsymbol{v}_2$

$$\begin{aligned} v_2 &= v - v_1 \\ &= (8i + 5j) - \left(-\frac{2}{5}i + \frac{4}{5}j \right) \\ &= (8i + 5j) + \left(\frac{2}{5}i - \frac{4}{5}j \right) \\ &= \left(8 + \frac{2}{5} \right) i + \left(5 - \frac{4}{5} \right) j \\ &= \frac{42}{5}i + \frac{21}{5}j \end{aligned}$$

Check

$$\begin{aligned} v_1 \cdot v_2 &= 0 \\ \left(-\frac{2}{5} \right)\left(\frac{42}{5} \right) + \left(\frac{4}{5} \right)\left(\frac{21}{5} \right) &= 0 \\ -\frac{84}{25} + \frac{84}{25} &= 0 \\ 0 &= 0 \end{aligned}$$

Exercise 5: A farmer is plowing a field with a mule pulling a plow. How much work is the mule doing pulling the plow the length of a field that is 275 feet long, if it is pulling the plow with a constant force of 240 lb and the straps make an angle of 28° to with the horizontal? Round answer to nearest tenth.

$$\begin{aligned} W &= \|F\| \left\| \overrightarrow{AB} \right\| \cos\theta \\ &= (240\ lb)(275\ \ feet) \cos 28^\circ \\ &\approx 58,274.5\ \ foot - pounds \end{aligned}$$

The amount of work done is approximately 58,274.5 foot-pounds.