# Review Exercise Set 10

Exercise 1: Use a sum or difference formula to find the exact value of the given expression.

$$\cos 183^\circ \cos 153^\circ + \sin 183^\circ \sin 153^\circ$$

Exercise 2: Verify the given identity.

$$\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

Exercise 3: Verify the given identity.

$$\frac{\sin(\alpha - \beta)}{\sin\alpha \cos\beta} = 1 - \cot\alpha \tan\beta$$

Exercise 4: Find the exact value of $\sin(\alpha+\beta)$ under the given conditions.

$$\sin\alpha = \frac{3}{5}\ \textit{and}\ \alpha\ \textit{lies in quadrant I};\ \sin\beta = \frac{12}{13}\ \textit{and}\ \beta\ \textit{lies in quadrant II}$$

Exercise 5: Find the exact value of $\tan(\alpha+\beta)$ under the given conditions.

$$\tan\alpha = -3\ \textit{and}\ \alpha\ \textit{lies in quadrant II};\ \cot\beta = -3\ \textit{and}\ \beta\ \textit{lies in quadrant IV}$$

# Review Exercise Set 10 Answer Key

Exercise 1: Use a sum or difference formula to find the exact value of the given expression.

$$\cos 183^\circ \cos 153^\circ + \sin 183^\circ \sin 153^\circ$$

Use the $\cos(\alpha - \beta)$ formula; $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

$$\begin{aligned} \cos 183^\circ \cos 153^\circ + \sin 183^\circ \sin 153^\circ &= \cos(183^\circ - 153^\circ) \\ &= \cos(30^\circ) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

Exercise 2: Verify the given identity.

$$\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

Use the $\cos(\alpha + \beta)$ formula; $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$$\begin{aligned} \cos\left(x + \frac{\pi}{4}\right) &= \frac{\sqrt{2}}{2}(\cos x - \sin x) \\ \cos(x)\cos\left(\frac{\pi}{4}\right) - \sin(x)\sin\left(\frac{\pi}{4}\right) &= \frac{\sqrt{2}}{2}(\cos x - \sin x) \end{aligned}$$

Evaluate cos and sin at $\frac{\pi}{4}$

$$\cos(x) \times \frac{\sqrt{2}}{2} - \sin(x) \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

Factor out common term

$$\frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{\sqrt{2}}{2}(\cos x - \sin x)$$

Exercise 3: Verify the given identity.

$$\frac{\sin(\alpha-\beta)}{\sin\alpha\cos\beta}=1-\cot\alpha\tan\beta$$

Use the $\sin(\alpha-\beta)$ formula in the numerator of the fraction

$$\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\sin\alpha\cos\beta}=1-\cot\alpha\tan\beta$$

Separate the fraction into two fractions

$$\frac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta}-\frac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta}=1-\cot\alpha\tan\beta$$

Simplify the fractions

$$1-\frac{\cos\alpha}{\sin\alpha}\times\frac{\sin\beta}{\cos\beta}=1-\cot\alpha\tan\beta$$

Use the quotient identities

$$1-\cot\alpha\tan\beta=1-\cot\alpha\tan\beta$$

Exercise 4: Find the exact value of $\sin(\alpha+\beta)$ under the given conditions.

$$\sin\alpha=\frac{3}{5}\ \textit{and}\ \alpha\ \textit{lies in quadrant I};\ \sin\beta=\frac{12}{13}\ \textit{and}\ \beta\ \textit{lies in quadrant II}$$

Find $\cos\alpha$

Since $\alpha$ lies in quadrant I cosine is also positive. Use the Pythagorean Theorem to find the length of the adjacent side.

$$\sin\alpha=\frac{3}{5}=\frac{y}{r}$$

$$\begin{aligned} x^2+y^2&=r^2\\ x^2+3^2&=5^2\\ x^2+9&=25\\ x^2&=16\\ x&=4 \end{aligned}$$

$$\cos\alpha=\frac{x}{r}=\frac{4}{5}$$

Exercise 4 (Continued):

Find $\cos\beta$

Since $\beta$ lies in quadrant II cosine must be negative. Use the Pythagorean Theorem to find the length of the adjacent side.

$$\sin\beta = \frac{12}{13} = \frac{y}{r}$$

$$\begin{aligned} x^2 + y^2 &= r^2 \\ x^2 + 12^2 &= 13^2 \\ x^2 + 144 &= 169 \\ x^2 &= 25 \\ x &= 5 \end{aligned}$$

$$\cos\beta = \frac{x}{r} = -\frac{5}{13}$$

Use the $\sin(\alpha+\beta)$ formula and substitute in the known values

$$\begin{aligned} \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \\ &= \left(\frac{3}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) \\ &= -\frac{15}{65} + \frac{48}{65} \\ &= \frac{33}{65} \end{aligned}$$

Exercise 5: Find the exact value of $\tan(\alpha+\beta)$ under the given conditions.

$\tan\alpha = -3$ *and* $\alpha$ *lies in quadrant II*; $\cot\beta = -3$ *and* $\beta$ *lies in quadrant IV*

Find $\tan(\beta)$

Since $\beta$ lies in quadrant IV tangent must be negative. Use the reciprocal identity to find $\tan(\beta)$.

$$\begin{aligned} \tan(\beta) &= \frac{1}{\cot(\beta)} \\ &= \frac{1}{-3} \\ &= -\frac{1}{3} \end{aligned}$$

Exercise 5 (Continued):

Use the $\tan(\alpha+\beta)$ formula and substitute in the known values

$$\begin{aligned}\tan(\alpha+\beta) &= \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}\\ &= \frac{-3-\frac{1}{3}}{1-(-3)\left(-\frac{1}{3}\right)}\\ &= \frac{-3-\frac{1}{3}}{1-1}\\ &= \frac{-3-\frac{1}{3}}{0}\\ &= \mathit{undefined}\end{aligned}$$