Review Exercise Set 12

Exercise 1: Express the given product as a sum or difference.

$$\sin(-4x)\sin(8x)$$

Exercise 2: Express the given sum or difference as a product.

$$\cos(9x)+\cos(4x)$$

Exercise 3: Find the exact value using product-to-sum identities.

$$2\cos 15^\circ \sin 135^\circ$$

Exercise 4: Verify the given identity.

$$\frac{\sin m+\sin n}{\cos m+\cos n}=\tan\frac{m+n}{2}$$

Exercise 5: Verify the given identity.

$$\frac{\cos x+\cos 3x}{\sin x+\sin 3x}=\cot 2x$$

Review Exercise Set 12 Answer Key

Exercise 1: Express the given product as a sum or difference.

$$\sin(-4x)\sin(8x)$$

Use the identity for the product of two sines

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$$

$$\sin(-4x)\sin(8x) = \frac{1}{2}\left[\cos(-4x-8x)-\cos(-4x+8x)\right]$$

$$= \frac{1}{2}\left[\cos(-12x)-\cos(4x)\right]$$

$$= \frac{1}{2}\cos(-12x)-\frac{1}{2}\cos(4x)$$

Exercise 2: Express the given sum or difference as a product.

$$\cos(9x)+\cos(4x)$$

Use the identity for the sum of two cosines

$$\cos\alpha+\cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos(9x)+\cos(4x) = 2\cos\frac{9x+4x}{2}\cos\frac{9x-4x}{2}$$

$$= 2\cos\frac{13x}{2}\cos\frac{5x}{2}$$

Exercise 3: Find the exact value using product-to-sum identities.

$$2\cos 15^\circ \sin 135^\circ$$

Apply the product to sum identity

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$

$$2\cos 15^\circ\sin 135^\circ = 2\left(\frac{1}{2}\left[\sin(15^\circ+135^\circ)-\sin(15^\circ-135^\circ)\right]\right)$$

Exercise 3 (Continued):

Simplify

$$2\cos 15^\circ \sin 135^\circ = \sin(150^\circ) - \sin(-120^\circ)$$

Apply the even/odd identity for sin (-x)

$$2\cos 15^\circ \sin 135^\circ = \sin(150^\circ) + \sin(120^\circ)$$

Rewrite in terms of the reference angles

$$\sin(150^\circ) = \sin(30^\circ) \quad \textit{and} \quad \sin(120^\circ) = \sin(60^\circ)$$

$$\begin{aligned} 2\cos 15^\circ \sin 135^\circ &= \sin(30^\circ) + \sin(60^\circ) \\ &= \frac{1}{2} + \frac{\sqrt{3}}{2} \\ &= \frac{1+\sqrt{3}}{2} \end{aligned}$$

Exercise 4: Verify the given identity.

$$\frac{\sin m + \sin n}{\cos m + \cos n} = \tan\frac{m+n}{2}$$

Use the sum of sines and sum of cosines identities

$$\frac{2\sin\frac{m+n}{2}\cos\frac{m-n}{2}}{2\cos\frac{m+n}{2}\cos\frac{m-n}{2}} = \tan\frac{m+n}{2}$$

Reduce common terms

$$\frac{\sin\frac{m+n}{2}}{\cos\frac{m+n}{2}} = \tan\frac{m+n}{2}$$

Use the quotient identity

$$\tan\frac{m+n}{2} = \tan\frac{m+n}{2}$$

Exercise 5: Verify the given identity.

$$\frac{\cos x + \cos 3x}{\sin x + \sin 3x} = \cot 2x$$

Use the sum of sines and cosines identities

$$\frac{2\cos\frac{x+3x}{2}\cos\frac{x-3x}{2}}{2\sin\frac{x+3x}{2}\cos\frac{x-3x}{2}} = \cot 2x$$

Reduce common terms

$$\frac{\cos 2x}{\sin 2x} = \cot 2x$$

Use the quotient identity

$$\cot 2x = \cot 2x$$