Review Exercise Set 13

Exercise 1: Find all real solutions to the equation $\sqrt{3}\tan x - 1 = 0$.

Exercise 2: Find all real solutions to the equation $6\cos(2x) = -3$.

Exercise 3: Find all solution in the interval [0, 2π) for the given equation.

$$2\cos x \sin x - \cos x = 0$$

Exercise 4: Find all solution in the interval [0, 2π) for the given equation.

$$2\sin^2 x + 7\sin x = 4$$

Exercise 5: Find all solution in the interval [0, 2π) for the given equation.

$$\cos(2x) + 2\sin^2 x - 3\sin x = 0$$

Review Exercise Set 13 Answer Key

Exercise 1: Find all real solutions to the equation $\sqrt{3}\tan x - 1 = 0$.

Isolate the function on one side of the equation

$$\sqrt{3}\tan x - 1 = 0$$
$$\sqrt{3}\tan x = 1$$
$$\tan x = \frac{1}{\sqrt{3}}$$
$$\tan x = \frac{\sqrt{3}}{3}$$

Identify the quadrants for the solutions on the interval of [0, π)

tangent is positive in quadrant I

Solve for x

$$x = \frac{\pi}{6}$$

Add nπ to the value of x

$$x = \frac{\pi}{6} + n\pi$$

Exercise 2: Find all real solutions to the equation $6\cos(2x) = -3$.

Isolate the function on one side of the equation

$$6\cos(2x) = -3$$
$$\cos(2x) = -\frac{1}{2}$$

Identify the quadrants for the solutions on the interval of [0, 2π)

cosine is negative in quadrants II and III

Exercise 2 (Continued):

Solve for the angle 2x

$\cos\frac{\pi}{3}=\frac{1}{2}$ so

$2x=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$ (quadrant II) and

$2x=\pi+\frac{\pi}{3}=\frac{4\pi}{3}$ (quadrant III)

Add 2nπ to the angle and solve for x

$$2x=\frac{2\pi}{3}+2n\pi$$
$$x=\frac{\pi}{3}+n\pi$$

and

$$2x=\frac{4\pi}{3}+2n\pi$$
$$x=\frac{2\pi}{3}+n\pi$$

Exercise 3: Find all solution in the interval [0, 2π) for the given equation.

$$2\cos x\sin x-\cos x=0$$

Factor the left side of the equation

$$\cos x\left(2\sin x-1\right)=0$$

Set each factor equal to zero

$$\cos x=0$$
$$x=\frac{\pi}{2}\ and\ \frac{3\pi}{2}$$

and

$$2\sin x-1=0$$
$$\sin x=\frac{1}{2}$$
$$x=\frac{\pi}{6}\ and\ \frac{5\pi}{6}$$

The solutions are $x=\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6},\ and\ \frac{3\pi}{2}$.

Exercise 4: Find all solution in the interval [0, 2π) for the given equation.

$$2\sin^2 x + 7\sin x = 4$$

Group all terms on the left side

$$2\sin^2 x + 7\sin x - 4 = 0$$

Let u represent the trigonometric function sin x

$$u = \sin x$$

$$2u^2 + 7u - 4 = 0$$

Factor the quadratic equation

$$(2u - 1)(u + 4) = 0$$

Solve for u

$$2u - 1 = 0 \qquad \text{or} \qquad u + 4 = 0$$

$$u = \frac{1}{2} \qquad\qquad u = -4$$

Substitute the sine function back in for u

$$\sin x = \frac{1}{2} \quad or \quad \sin x = -4$$

sin x must be between -1 and 1 so sin x = -4 has no solution

Solve for x

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}\, or\, \frac{5\pi}{6}$$

The solutions are $\frac{\pi}{6}\, or\, \frac{5\pi}{6}$.

Exercise 5: Find all solution in the interval [0, 2π) for the given equation.

$$\cos(2x) + 2\sin^2 x - 3\sin x = 0$$

Use the cosine double angle identity

$$1 - 2\sin^2 x + 2\sin^2 x - 3\sin x = 0$$
$$1 - 3\sin x = 0$$

Isolate the function on one side of the equation

$$\sin x = \frac{1}{3}$$

Identify the quadrants for the solutions on the interval of [0, 2π)

sine is positive in quadrants I and II

Solve for x

x ≈ 0.3398 (quadrant I)

x ≈ π - 0.3398 ≈ 2.8018 (quadrant II)