Review Exercise Set 13

Exercise 1: Find all real solutions to the equation $\sqrt{3} \tan x - 1 = 0$.

Exercise 2: Find all real solutions to the equation $6 \cos (2x) = -3$.

Exercise 3: Find all solutions in the interval $[0, 2\pi)$ for the given equation.

$$2 \cos x \sin x - \cos x = 0$$
Exercise 4: Find all solution in the interval \([0, 2\pi]\) for the given equation.

\[2 \sin^2 x + 7 \sin x = 4\]

Exercise 5: Find all solution in the interval \([0, 2\pi]\) for the given equation.

\[\cos(2x) + 2 \sin^2 x - 3 \sin x = 0\]
Review Exercise Set 13 Answer Key

Exercise 1: Find all real solutions to the equation $\sqrt{3} \tan x - 1 = 0$.

Isolate the function on one side of the equation

$$\sqrt{3} \tan x - 1 = 0$$
$$\sqrt{3} \tan x = 1$$
$$\tan x = \frac{1}{\sqrt{3}}$$

Identify the quadrants for the solutions on the interval of $[0, \pi)$

- tangent is positive in quadrant I

Solve for $x$

$$x = \frac{\pi}{6}$$

Add $n\pi$ to the value of $x$

$$x = \frac{\pi}{6} + n\pi$$

Exercise 2: Find all real solutions to the equation $6 \cos (2x) = -3$.

Isolate the function on one side of the equation

$$6 \cos (2x) = -3$$
$$\cos (2x) = -\frac{1}{2}$$

Identify the quadrants for the solutions on the interval of $[0, 2\pi)$

- cosine is negative in quadrants II and III
Exercise 2 (Continued):

Solve for the angle $2x$

\[
\cos \frac{\pi}{3} = \frac{1}{2} \quad \text{so}
\]

\[
2x = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \quad \text{(quadrant II) and}
\]

\[
2x = \pi + \frac{\pi}{3} = \frac{4\pi}{3} \quad \text{(quadrant III)}
\]

Add $2n\pi$ to the angle and solve for $x$

\[
2x = \frac{2\pi}{3} + 2n\pi \quad \text{and} \quad 2x = \frac{4\pi}{3} + 2n\pi
\]

\[
x = \frac{\pi}{3} + n\pi
\]

Exercise 3: Find all solutions in the interval $[0, 2\pi)$ for the given equation.

\[
2 \cos x \sin x - \cos x = 0
\]

Factor the left side of the equation

\[
\cos x (2 \sin x - 1) = 0
\]

Set each factor equal to zero

\[
\cos x = 0 \quad \text{and} \quad 2 \sin x - 1 = 0
\]

\[
x = \frac{\pi}{2} \quad \text{and} \quad \frac{3\pi}{2} \quad \text{and} \quad \sin x = \frac{1}{2}
\]

\[
\frac{\pi}{6} \quad \text{and} \quad \frac{5\pi}{6}
\]

The solutions are $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \text{and} \frac{3\pi}{2}$. 
Exercise 4: Find all solutions in the interval \([0, 2\pi]\) for the given equation.

\[ 2 \sin^2 x + 7 \sin x = 4 \]

Group all terms on the left side

\[ 2 \sin^2 x + 7 \sin x - 4 = 0 \]

Let \( u \) represent the trigonometric function \( \sin x \)

\[ u = \sin x \]

\[ 2u^2 + 7u - 4 = 0 \]

Factor the quadratic equation

\[ (2u - 1)(u + 4) = 0 \]

Solve for \( u \)

\[ 2u - 1 = 0 \quad \text{or} \quad u + 4 = 0 \]

\[ u = \frac{1}{2} \quad \text{or} \quad u = -4 \]

Substitute the sine function back in for \( u \)

\[ \sin x = \frac{1}{2} \quad \text{or} \quad \sin x = -4 \]

\( \sin x \) must be between -1 and 1 so \( \sin x = -4 \) has no solution

Solve for \( x \)

\[ \sin x = \frac{1}{2} \]

\[ x = \frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6} \]

The solutions are \( \frac{\pi}{6} \) or \( \frac{5\pi}{6} \).
Exercise 5: Find all solution in the interval \([0, 2\pi]\) for the given equation.

\[
\cos(2x) + 2\sin^2 x - 3\sin x = 0
\]

Use the cosine double angle identity

\[
1 - 2\sin^2 x + 2\sin^2 x - 3\sin x = 0
\]

\[
1 - 3\sin x = 0
\]

Isolate the function on one side of the equation

\[
\sin x = \frac{1}{3}
\]

Identify the quadrants for the solutions on the interval of \([0, 2\pi]\)

sine is positive in quadrants I and II

Solve for \(x\)

\[
x \approx 0.3398 \text{ (quadrant I)}
\]

\[
x \approx \pi - 0.3398 \approx 2.8018 \text{ (quadrant II)}
\]