Review Exercise Set 9

Exercise 1: Verify the given identity.

$$\tan x\left(\cot x+\tan x\right)=\sec^2 x$$

Exercise 2: Verify the given identity.

$$\frac{1+\sin x}{\cos^2 x}=\tan^2 x+1+\tan x\sec x$$

Exercise 3: Verify the given identity.

$$\cos x+\sin x\tan x=\sec x$$

Exercise 4: Verify the given identity.

$$\sec^2 x \cot^2 x = \csc^2 x$$

Exercise 5: Verify the given identity.

$$\frac{1}{\sin x + \cos x} + \frac{1}{\sin x - \cos x} = \frac{2\sin x}{\sin^2 x - \cos^2 x}$$

Review Exercise Set 9 Answer Key

Exercise 1: Verify the given identity.

$$\tan x\left(\cot x+\tan x\right)=\sec^2 x$$

Distribute the tan x on the left side of the equation

$$\tan x\cot x+\tan^2 x=\sec^2 x$$

Use the quotient identities to rewrite tan x and cot x

$$\frac{\sin x}{\cos x}\times\frac{\cos x}{\sin x}+\tan^2 x=\sec^2 x$$
$$1++\tan^2 x=\sec^2 x$$

Use the Pythagorean identity for $1+\tan^2 x$

$$\sec^2 x=\sec^2 x$$

Exercise 2: Verify the given identity.

$$\frac{1+\sin x}{\cos^2 x}=\tan^2 x+1+\tan x\sec x$$

Separate the fraction on the left side into two fractions

$$\frac{1+\sin x}{\cos^2 x}=\tan^2 x+1+\tan x\sec x$$
$$\frac{1}{\cos^2 x}+\frac{\sin x}{\cos^2 x}=\tan^2 x+1+\tan x\sec x$$

Rewrite the fractions into simpler forms

$$\left(\frac{1}{\cos x}\right)^2+\frac{\left(\sin x\right)\times 1}{\cos x\times\cos x}=\tan^2 x+1+\tan x\sec x$$
$$\left(\frac{1}{\cos x}\right)^2+\frac{\sin x}{\cos x}\times\frac{1}{\cos x}=\tan^2 x+1+\tan x\sec x$$

Exercise 2 (Continued):

Use the reciprocal and quotient identities

$$(\sec x)^2 + \tan x \sec x = \tan^2 x + 1 + \tan x \sec x$$
$$\sec^2 x + \tan x \sec x = \tan^2 x + 1 + \tan x \sec x$$

Use the Pythagorean identity for $\sec^2$ x

$$\tan^2 x + 1 + \tan x \sec x = \tan^2 x + 1 + \tan x \sec x$$

Exercise 3: Verify the given identity.

$$\cos x + \sin x \tan x = \sec x$$

Use the quotient identity for tan x

$$\cos x + \sin x \times \frac{\sin x}{\cos x} = \sec x$$
$$\cos x + \frac{\sin^2 x}{\cos x} = \sec x$$

Use the Pythagorean identity $\sin^2$ x

$$\cos x + \frac{1 - \cos^2 x}{\cos x} = \sec x$$

Reduce the fraction

$$\cos x + \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \sec x$$
$$\cos x + \frac{1}{\cos x} - \cos x = \sec x$$

Combine like terms

$$\frac{1}{\cos x} = \sec x$$

Use the reciprocal identity for sec x

$$\sec x = \sec x$$

Exercise 4: Verify the given identity.

$$\sec^2 x \cot^2 x = \csc^2 x$$

Use the quotient and reciprocal identities

$$\left(\frac{1}{\cos x}\right)^2 \left(\frac{\cos x}{\sin x}\right)^2 = \csc^2 x$$

Simplify

$$\left(\frac{1}{\cos x} \times \frac{\cos x}{\sin x}\right)^2 = \csc^2 x$$

$$\left(\frac{1}{\sin x}\right)^2 = \csc^2 x$$

Use the reciprocal identity for csc x

$$(\csc x)^2 = \csc^2 x$$

$$\csc^2 x = \csc^2 x$$

Exercise 5: Verify the given identity.

$$\frac{1}{\sin x + \cos x} + \frac{1}{\sin x - \cos x} = \frac{2\sin x}{\sin^2 x - \cos^2 x}$$

Combine the fractions on the left side by getting common denominators

$$\frac{1}{\sin x + \cos x} \times \frac{\sin x - \cos x}{\sin x - \cos x} + \frac{1}{\sin x - \cos x} \times \frac{\sin x + \cos x}{\sin x + \cos x} = \frac{2\sin x}{\sin^2 x - \cos^2 x}$$

$$\frac{\sin x - \cos x}{\sin^2 x - \cos^2 x} + \frac{\sin x + \cos x}{\sin^2 x - \cos^2 x} = \frac{2\sin x}{\sin^2 x - \cos^2 x}$$

$$\frac{\sin x - \cos x + \sin x + \cos x}{\sin^2 x - \cos^2 x} = \frac{2\sin x}{\sin^2 x - \cos^2 x}$$

Combine like terms

$$\frac{2\sin x}{\sin^2 x - \cos^2 x} = \frac{2\sin x}{\sin^2 x - \cos^2 x}$$