VERIFYING TRIGONOMETRIC IDENTITIES

Basic Identities Review

- The basic trigonometric identities consist of the reciprocal identities, quotient identities, identities for negatives, and the Pythagorean identities.
- These identities were introduced in Chapter 5 Section 2, however in this chapter we are going to review the basic identities and show how to use them to determine other identities.
- The following slides consist of the basic identities summarized individually. These should be memorized because they are going to be often used in the problems that follow.

Reciprocal Identities

- **csc x = 1/ sin x**

  Example: sin x = $-\frac{1}{2}$
  Answer: csc x = $1/(-\frac{1}{2})$
  = $1(-2/1)$
  = **-2**

- **sec x = 1/ cos x**

  Example: cos x = $-\sqrt{2}/2$
  Answer: sec x = $1/ -\sqrt{2}/2$
  = $1(-2/\sqrt{2})$
  = **$-2/\sqrt{2}$**

- **cot x = 1/ tan x**

  Example: sin x = $-\frac{1}{2}$, and cos x = $-\sqrt{2}/2$
  Answer: cot x = 1/ tan x
  = 1/ (sin x/cos x)
  = $1/ [(-\frac{1}{2})/ (-\sqrt{2}/2)]$
  = $1/ [(-\frac{1}{2})(-2/\sqrt{2})]$
  = $1/ (1/\sqrt{2}) =$ **$\sqrt{2}$**

Quotient Identities

- **tan x = sin x/ cos x**

  Example: sin x = 0, and cos x = -1
  Answer: tan x = 0/-1
  = **0**

- **cot x = cos x/ sin x**

  Example: cos x = $\frac{1}{2}$, and sin x = $\sqrt{3}/2$
  Answer: cot x = $\frac{1}{2} / \sqrt{3}/2$
  = $\frac{1}{2}(2/\sqrt{3})$
  = **$1/\sqrt{3}$**

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Identities for Negatives

- sin (-x) = -sin x
- cos (-x) = cos x
- tan (-x) = -tan x

Pythagorean Identity

$\sin^2 x + \cos^2 x = 1$

Example: $\sin x = \sqrt{3}/2$
Answer: $(\sqrt{3}/2)^2 + \cos^2 x = 1$
$\frac{3}{4} + \cos^2 x = 1$
$\cos^2 x = 1 - \frac{3}{4}$
$\sqrt{\cos^2 x} = (\sqrt{1/4})$
$\cos x = \frac{1}{2}$

These Pythagorean Identities are also included:

$\tan^2 x + 1 = \sec^2 x$ **$1 + \cot^2 x = \csc^2 x$**

Establishing Other Identities

- To verify an identity equals to the other, many steps are taken to prove that both sides of the equation are equal to each other.
- To prove both sides are equal to each other, we will use basic identities, algebra, and other justified identities.
- Now, I must express that on many problems there are several ways to find the solutions. In other words, to prove both sides of the equation, various identities and algebraic operations can be used to confirm they are equal.
- The examples demonstrated are just that, examples. Just because the example was proven one way does not mean that is the only way, there can be other ways.

Identity Verification

(cos x)(tan x) = sin x

Step 1 Pick the most complicated of both sides, in this case **(cos x)(tan x)**

Step 2 Transform **(cos x)(tan x)** into **sin x** by using identities and algebraic operations.

Here it is step-by-step:

(cos x)(tan x) = (cos x)(sin x/cos x) (quotient identity)
= **sin x** (algebra, both cos x were cancelled)

Example 1: **(sin x)(sec x) = tan x**

Answer:

(sin x)(sec x)	= sin x (1/cos x)	[reciprocal identity]
	= sin x/cos x	[algebra]
	= tan x	[quotient identity]

Example 2: **sin (-x)/ cos (-x) = - tan x**

Answer:

sin (-x) / cos (-x)	= -sin x/ cos x	[identity for negatives]
	= -tan x	[quotient identity]

Example 3: **sin x = [(tan x)(cot x)]/ csc x**

Answer:

[(tan x)(cot x)]/csc x	= [(sin x/cos x)(cos x/sin x)]/(1/sin x)	[quotient & reciprocal identity]
	= 1/ (1/sin x)	[algebra, both sin x and cos x were cancelled]
	= 1 (sin x/1)	[algebra, multiplication]
	= sin x	

Example 4: **$\sin^2 x/\cos x + \cos x = \sec x$**

Answer:

$\sin^2 x/\cos x + \cos x$	$= \sin^2 x/\cos x + (\cos x)(\cos x/\cos x)$	[algebra, found common denominator cos x]
	$= [\sin^2 x + \cos^2 x]/\cos x$	
	$= 1/\cos x$	[Pythagorean identity]
	$= \sec x$	[reciprocal identity]

Key Suggestions

- Looking at others do the work or just following numerous examples, does not guarantee that you will be good at verifying identities. Doing and verifying on your own, makes it less complicated and you will have more confidence as you progress.
- Regarding algebraic operations, use methods such as, but limited to, multiplying, factoring, splitting fractions, combining fractions, and cancellations.
- If you are stuck, try to denote each function as sine and cosine functions and do proper algebra.
- Try not to forget the other side of the equation. Treat it as a goal that needs to be reached ultimately, so keep it in mind.

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