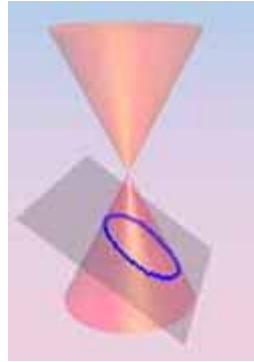


# ELLIPSE

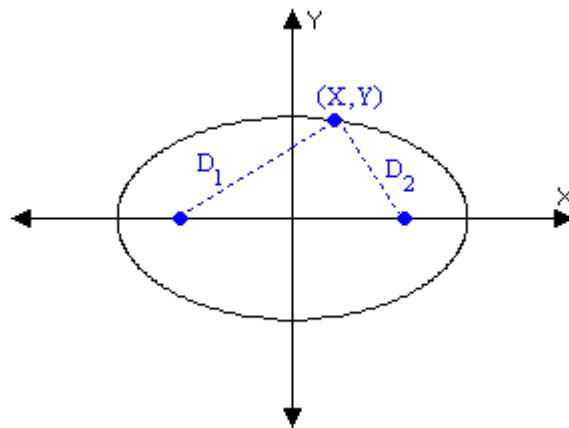
This section will discuss the conic section known as an ellipse. An ellipse is created when a plane cuts completely through the cone and is not perpendicular to the cones axis (see picture)



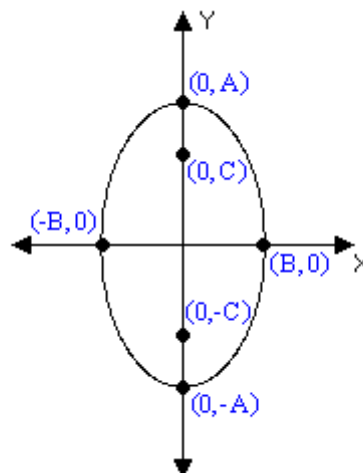
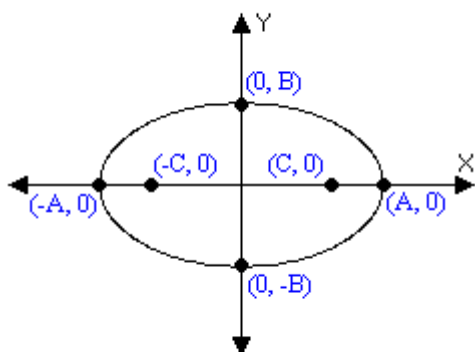
and is defined as:

*a set of all points the sum of whose distances from distinct fixed points (foci) is constant.*

The picture below illustrates this.



A line that passes through both foci and intersects the ellipse at two points is known as the major axis. The midpoint of this chord is known as the center. A chord that is perpendicular to the major axis and passes through the center is known as the minor axis. Note that the major axis may be horizontal or vertical depending on the location of the foci. The illustration below represents this idea.



Because the major axis may be horizontal or vertical, the standard form of the equation of an ellipse is made up of two forms when the center is at the origin.

**Major axis horizontal**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, 0 < b < a$$

**Major axis vertical**

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, 0 < b < a$$

The foci for an ellipse is found by using the formula  $c^2 = a^2 - b^2$ . The lengths of the axis are given by:

**Major axis**

$$2a$$

**Minor axis**

$$2b$$

The following examples will show how to use these formulas when solving for problems involving ellipses.

**Example 1:** Find the standard form of the equation of the ellipse that has a major axis of length 6 and foci at  $(-2, 0)$  and  $(2, 0)$  and center at the origin. Graph.

Solution:

Step 1: Analysis.

Since the foci are on the x axis and the ellipse has a center at the origin, the major axis is horizontal. The formula to be used will be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, 0 < b < a$$

**Example 1 (Continued):**

Step 2: Solve for  $a$ .

Since the length of the major axis is given as 6,  $a$  is solved for by using the length formula:

$$2a = 6$$

$$a = 3$$

Step 3: Solve for  $b$ .

Since the center is at  $(0, 0)$ , the distance to either foci  $c$  is 2. The foci formula is used then to solve for  $b$ .

$$c^2 = a^2 - b^2$$

$$(2)^2 = (3)^2 - b^2$$

$$b^2 = 9 - 4 = 5$$

$$b = \sqrt{5}$$

Step 4: Substitute.

The values found are substituted onto the formula to find the equation.

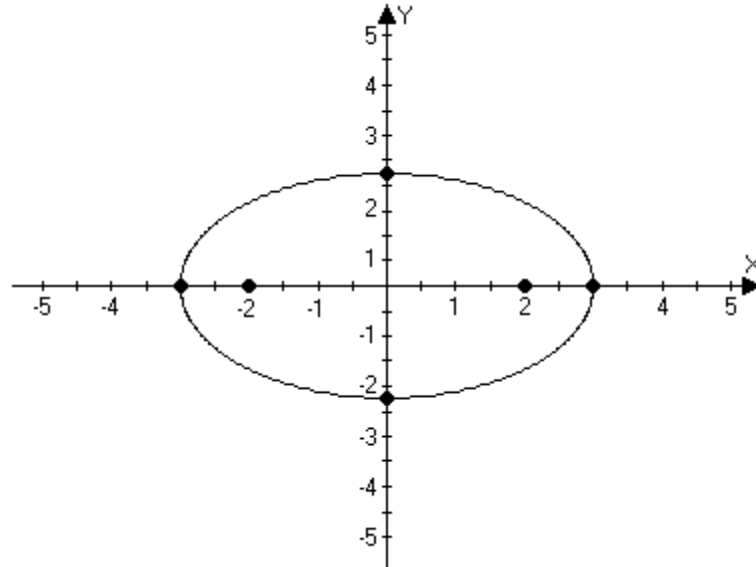
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{(3)^2} + \frac{y^2}{(\sqrt{5})^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

**Example 1 (Continued):**

Step 5: Plot and graph.



**Example 2:** Sketch the ellipse given by  $4x^2 + y^2 = 36$  and identify the vertices.

Solution:

Step 1: Rewrite the equation in standard form.

The given equation is rewritten into the standard form  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  .

$$4x^2 + y^2 = 36$$

$$\frac{4x^2}{36} + \frac{y^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

$$\frac{x^2}{3^2} + \frac{y^2}{6^2} = 1$$

**Example 2 (Continued):**

Step 2: Analysis.

Since the  $y^2$  denominator term is larger of the two, the major axis is vertical.

Since  $a = 6$ , the vertices are  $(0, -6)$  and  $(0, 6)$ .

Since  $b = 3$ , the endpoints of the minor axis are  $(-3, 0)$  and  $(3, 0)$ .

Since  $c^2 = a^2 - b^2 = 25$  the foci are  $(0, -5)$  and  $(0, 5)$ .

Step 3: Graph.

The values found in step 3 are plotted to create the graph.

