Review Exercise Set 21

Exercise 1: Graph the given ellipse and locate its foci.

\[ \frac{x^2}{25} + \frac{y^2}{9} = 1 \]

Exercise 2: Find the equation of an ellipse in standard form with foci at (-1, 0) and (1, 0) and vertices at (-3, 0) and (3, 0).

Exercise 3: Graph the given ellipse centered at (h, k) and locate its foci.

\[ \frac{(x+3)^2}{25} + \frac{(y-1)^2}{9} = 1 \]
Exercise 4: Convert the given equation into the standard form of an ellipse by completing the square of $x$ and $y$.

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

Exercise 5: Will a truck driver be able to safely drive his truck, which is 10 feet wide and reaches a maximum height above the ground of 12 feet, under a bridge with an elliptical archway, which has a maximum height (at the center of the road) of 15 feet and a width of 40 feet, without going into the oncoming traffic lane?
Exercise 1: Graph the given ellipse and locate its foci.

\[ \frac{x^2}{25} + \frac{y^2}{9} = 1 \]

Identify the direction of the major axis

The major axis will be horizontal since the denominator of the \( x^2 \) term is greater than the denominator of the \( y^2 \) term.

Find the endpoints of the major and minor axes

\[ a^2 = 25 \]
\[ a = 5 \]

The endpoints (vertices) of the major axis \((-a, 0)\) and \((a, 0)\) are \((-5, 0)\) and \((5, 0)\)

\[ b^2 = 9 \]
\[ b = 3 \]

The endpoints of the minor axis \((0, -b)\) and \((0, b)\) are \((0, -3)\) and \((0, 3)\)

Find the foci

\[ c^2 = a^2 - b^2 \]
\[ c^2 = 25 - 9 \]
\[ c^2 = 16 \]
\[ c = 4 \]

The foci \((-c, 0)\) and \((c, 0)\) are \((-4, 0)\) and \((4, 0)\)

Sketch the graph
Exercise 2: Find the equation of an ellipse in standard form with foci at (-1, 0) and (1, 0) and vertices at (-3, 0) and (3, 0).

Identify the direction of the major axis

Since the foci are located on the x-axis the major axis will be horizontal.

Find the value of $a^2$

The vertices are at (-3, 0) and (3, 0) so the value of $a = 3$

$$a^2 = 3^2$$
$$a^2 = 9$$

Find the value of $b^2$

The foci are at (-1, 0) and (1, 0) so the value of $c = 1$

$$c^2 = a^2 - b^2$$
$$1^2 = 9 - b^2$$
$$b^2 = 9 - 1$$
$$b^2 = 8$$

Substitute the values of $a^2$ and $b^2$ into the equation of an ellipse

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

Exercise 3: Graph the given ellipse centered at $(h, k)$ and locate its foci.

$$\frac{(x + 3)^2}{25} + \frac{(y - 1)^2}{9} = 1$$

Locate the center $(h, k)$

$$\frac{(x - (-3))^2}{25} + \frac{(y - 1)^2}{9} = 1$$

$(h, k) = (-3, 1)$
Exercise 3 (Continued):

Identify the direction of the major axis

The major axis will be horizontal since the denominator of the $x^2$ term is greater than the denominator of the $y^2$ term.

Find the endpoints of the major and minor axes

\[ a^2 = 25 \]
\[ a = 5 \]

The endpoints of the major axis $(h - a, k)$ and $(h + a, k)$ are:

\[ (-3 - 5, 1) = (-8, 1) \quad \text{and} \quad (-3 + 5, 1) = (2, 1) \]

\[ b^2 = 9 \]
\[ b = 3 \]

The endpoints of the minor axis $(h, k - b)$ and $(h, k + b)$ are:

\[ (-3, 1 - 3) = (-3, -2) \quad \text{and} \quad (-3, 1 + 3) = (-3, 4) \]

Find the foci

\[ c^2 = a^2 - b^2 \]
\[ c^2 = 25 - 9 \]
\[ c^2 = 16 \]
\[ c = 4 \]

The foci $(h - c, k)$ and $(h + c, k)$ are:

\[ (-3 - 4, 1) = (-7, 1) \quad \text{and} \quad (-3 + 4, 1) = (1, 1) \]

Sketch the graph
Exercise 4: Convert the given equation into the standard form of an ellipse by completing the square of x and y.

\[9x^2 + 4y^2 - 18x + 16y - 11 = 0\]

Rewrite the equation grouping the x-terms and y-terms on the left and the constant on the right

\[(9x^2 - 18x) + (4y^2 + 16y) = 11\]

Factor so that the coefficients of the \(x^2\) and \(y^2\) terms is 1

\[9(x^2 - 2x) + 4(y^2 + 4y) = 11\]

Complete the square and simplify the equation

\[9\left(x^2 - 2x + \left(-\frac{2}{2}\right)^2\right) + 4\left(y^2 + 4y + \left(\frac{4}{2}\right)^2\right) = 11 + 9\left(-\frac{2}{2}\right)^2 + 4\left(\frac{4}{2}\right)^2\]

\[9\left(x - 1\right)^2 + 4\left(y + 2\right)^2 = 11 + 9 + 16\]

\[9\left(x - 1\right)^2 + 4\left(y + 2\right)^2 = 36\]

\[\frac{9\left(x - 1\right)^2}{36} + \frac{4\left(y + 2\right)^2}{36} = \frac{36}{36}\]

\[\frac{\left(x - 1\right)^2}{4} + \frac{\left(y + 2\right)^2}{9} = 1\]

Exercise 5: Will a truck driver be able to safely drive his truck, which is 10 feet wide and reaches a maximum height above the ground of 12 feet, under a bridge with an elliptical archway, which has a maximum height (at the center of the road) of 15 feet and a width of 40 feet, without going into the oncoming traffic lane?

Draw diagram of the problem
Exercise 5 (Continued):

Setup the equation for the elliptical archway

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]

\[
\frac{x^2}{20^2} + \frac{y^2}{15^2} = 1
\]

\[
\frac{x^2}{400} + \frac{y^2}{225} = 1
\]

Determine the clearance height when 10 feet from the center

\[
x = 10
\]

\[
\frac{10^2}{400} + \frac{y^2}{225} = 1
\]

\[
\frac{100}{400} + \frac{y^2}{225} = 1
\]

\[
\frac{1}{4} + \frac{y^2}{225} = 1
\]

\[
\frac{y^2}{225} = 1 - \frac{1}{4}
\]

\[
\frac{y^2}{225} = \frac{3}{4}
\]

\[
y^2 = \frac{3 \times 225}{4}
\]

\[
y = \sqrt{\frac{675}{4}}
\]

\[
y \approx 13
\]

Compare the height of the truck and the clearance height when \(x = 10\)

The truck is 12 feet high and the maximum clearance 10 feet from the center of the road is 13 feet, so the truck can safely travel under the bridge.