Review Exercise Set 21

Exercise 1: Graph the given ellipse and locate its foci.

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

Exercise 2: Find the equation of an ellipse in standard form with foci at (-1, 0) and (1, 0) and vertices at (-3, 0) and (3, 0).

Exercise 3: Graph the given ellipse centered at (h, k) and locate its foci.

$$\frac{(x+3)^2}{25}+\frac{(y-1)^2}{9}=1$$

Exercise 4: Convert the given equation into the standard form of an ellipse by completing the square of x and y.

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

Exercise 5: Will a truck driver be able to safely drive his truck, which is 10 feet wide and reaches a maximum height above the ground of 12 feet, under a bridge with an elliptical archway, which has a maximum height (at the center of the road) of 15 feet and a width of 40 feet, without going into the oncoming traffic lane?

Review Exercise Set 21 Answer Key

Exercise 1: Graph the given ellipse and locate its foci.

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

Identify the direction of the major axis

The major axis will be horizontal since the denominator of the x^2 term is greater than the denominator of the y^2 term.

Find the endpoints of the major and minor axes

$a^2 = 25$
$a = 5$

The endpoints (vertices) of the major axis (-a, 0) and (a, 0) are (-5, 0) and (5, 0)

$b^2 = 9$
$b = 3$

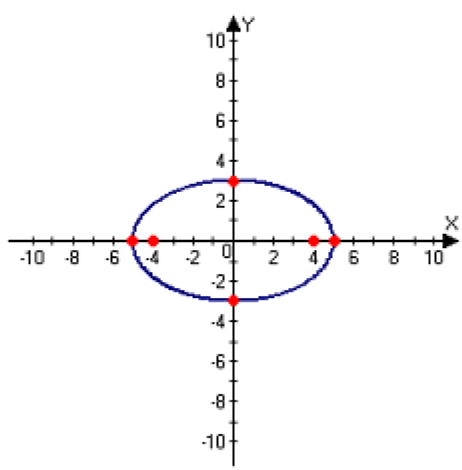
The endpoints of the minor axis (0, -b) and (0, b) are (0, -3) and (0, 3)

Find the foci

$c^2 = a^2 - b^2$
$c^2 = 25 - 9$
$c^2 = 16$
$c = 4$

The foci (-c, 0) and (c, 0) are (-4, 0) and (4, 0)

Sketch the graph



Exercise 2: Find the equation of an ellipse in standard form with foci at (-1, 0) and (1, 0) and vertices at (-3, 0) and (3, 0).

Identify the direction of the major axis

Since the foci are located on the x-axis the major axis will be horizontal.

Find the value of a^2

The vertices are at (-3, 0) and (3, 0) so the value of a = 3

$a^2 = 3^2$
$a^2 = 9$

Find the value of b^2

The foci are at (-1, 0) and (1, 0) so the value of c = 1

$c^2 = a^2 - b^2$
$1^2 = 9 - b^2$
$b^2 = 9 - 1$
$b^2 = 8$

Substitute the values of a^2 and b^2 into the equation of an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

Exercise 3: Graph the given ellipse centered at (h, k) and locate its foci.

$$\frac{(x+3)^2}{25} + \frac{(y-1)^2}{9} = 1$$

Locate the center (h, k)

$$\frac{(x-(-3))^2}{25} + \frac{(y-1)^2}{9} = 1$$

(h, k) = (-3, 1)

Exercise 3 (Continued):

Identify the direction of the major axis

The major axis will be horizontal since the denominator of the x^2 term is greater than the denominator of the y^2 term.

Find the endpoints of the major and minor axes

$a^2 = 25$
$a = 5$

The endpoints of the major axis (h - a, k) and (h + a, k) are:

(-3 - 5, 1) = (-8, 1) and (-3 + 5, 1) = (2, 1)

$b^2 = 9$
$b = 3$

The endpoints of the minor axis (h, k - b) and (h, k + b) are:

(-3, 1 - 3) = (-3, -2) and (-3, 1 + 3) = (-3, 4)

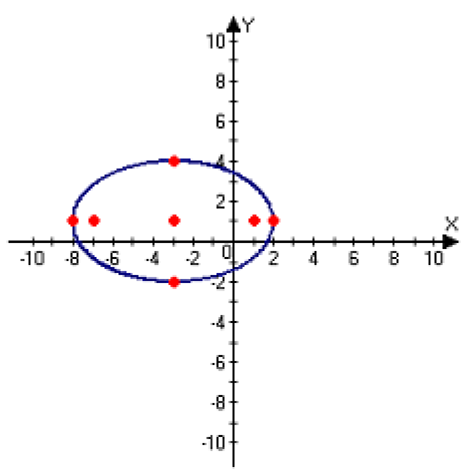
Find the foci

$c^2 = a^2 - b^2$
$c^2 = 25 - 9$
$c^2 = 16$
$c = 4$

The foci (h - c, k) and (h + c, k) are:

(-3 - 4, 1) = (-7, 1) and (-3 + 4, 1) = (1, 1)

Sketch the graph



Exercise 4: Convert the given equation into the standard form of an ellipse by completing the square of x and y.

$$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

Rewrite the equation grouping the x-terms and y-terms on the left and the constant on the right

$$(9x^2 - 18x) + (4y^2 + 16y) = 11$$

Factor so that the coefficients of the x^2 and y^2 terms is 1

$$9(x^2 - 2x) + 4(y^2 + 4y) = 11$$

Complete the square and simplify the equation

$$9\left(x^2 - 2x + \left(\frac{-2}{2}\right)^2\right) + 4\left(y^2 + 4y + \left(\frac{4}{2}\right)^2\right) = 11 + 9\left(\frac{-2}{2}\right)^2 + 4\left(\frac{4}{2}\right)^2$$

$$9\left(x^2 - 2x + (-1)^2\right) + 4\left(y^2 + 4y + (2)^2\right) = 11 + 9(-1)^2 + 4(2)^2$$

$$9(x-1)^2 + 4(y+2)^2 = 11 + 9 + 16$$

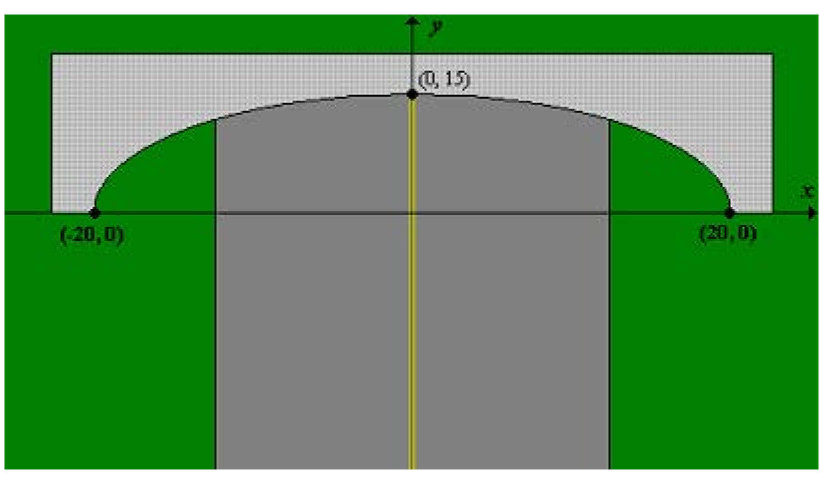
$$9(x-1)^2 + 4(y+2)^2 = 36$$

$$\frac{9(x-1)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

Exercise 5: Will a truck driver be able to safely drive his truck, which is 10 feet wide and reaches a maximum height above the ground of 12 feet, under a bridge with an elliptical archway, which has a maximum height (at the center of the road) of 15 feet and a width of 40 feet, without going into the oncoming traffic lane?

Draw diagram of the problem



Exercise 5 (Continued):

Setup the equation for the elliptical archway

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$
$$\frac{x^2}{20^2}+\frac{y^2}{15^2}=1$$
$$\frac{x^2}{400}+\frac{y^2}{225}=1$$

Determine the clearance height when 10 feet from the center

x = 10

$$\frac{10^2}{400}+\frac{y^2}{225}=1$$
$$\frac{100}{400}+\frac{y^2}{225}=1$$
$$\frac{1}{4}+\frac{y^2}{225}=1$$
$$\frac{y^2}{225}=1-\frac{1}{4}$$
$$\frac{y^2}{225}=\frac{3}{4}$$
$$y^2=\frac{3\times 225}{4}$$
$$y=\sqrt{\frac{675}{4}}$$
$$y\approx 13$$

Compare the height of the truck and the clearance height when x = 10

The truck is 12 feet high and the maximum clearance 10 feet from the center of the road is 13 feet, so the truck can safely travel under the bridge.