Review Exercise Set 22

Exercise 1: Find the vertices and foci for the given hyperbola. Sketch the graph.

$$\frac{y^2}{9} - \frac{x^2}{25} = 1$$

Exercise 2: Find the equation of a hyperbola in standard form with foci at (0, -5) and (0, 5) and vertices at (0, -2) and (0, 2).

Exercise 3: Find the foci and equations of the asymptotes.

$$\frac{x^2}{16} - \frac{y^2}{36} = 1$$

Exercise 4: Convert the given equation into the standard form of a hyperbola by completing the square of x and y. Find the foci and equations of the asymptotes. Sketch the graph.

$$9y^2 - x^2 + 54y + 4x + 68 = 0$$

Exercise 5: The perpendicular cross section of a nuclear power station cooling tower form two branches of a hyperbola. Suppose the central cross section of the tower is modeled by the hyperbola $2500x^2 - 625(y - 60)^2 = 1{,}000{,}000$ (x and y are in feet). What is the minimum distance between the sides of the tower?

Review Exercise Set 22 Answer Key

Exercise 1: Find the vertices and foci for the given hyperbola.

$$\frac{y^2}{9} - \frac{x^2}{25} = 1$$

Identify the transverse axis

Since the y^2 term is preceded by a plus sign, the transverse axis will be along the y-axis and the hyperbola would follow the equation of:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Find the vertices

$a^2 = 9$
$a = 3$

The vertices (0, -a) and (0, a) are (0, -3) and (0, 3)

Find the foci

$c^2 = a^2 + b^2$
$c^2 = 9 + 25$
$c^2 = 34$
$c = \sqrt{34}$

The foci (0, -c) and (0, c) are (0, -$\sqrt{34}$) and (0, $\sqrt{34}$)

Exercise 2: Find the equation of a hyperbola in standard form with foci at (0, -5) and (0, 5) and vertices at (0, -2) and (0, 2).

Identify the transverse axis

Since the foci are located on the y-axis, the transverse axis will be along the y-axis and the hyperbola would follow the equation of:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Exercise 2 (Continued):

Find the value of a^2

The vertices are at (0, -2) and (0, 2) so the value of a = 2

$a^2 = 2^2$
$a^2 = 4$

Find the value of b^2

The foci are at (0, -5) and (0, 5) so the value of c = 5

$c^2 = a^2 + b^2$
$5^2 = 4 + b^2$
$25 - 4 = b^2$
$b^2 = 21$

Substitute the values of a^2 and b^2 into the equation of a hyperbola

$$\frac{y^2}{4} - \frac{x^2}{21} = 1$$

Exercise 3: Find the foci and equations of the asymptotes.

$$\frac{x^2}{16} - \frac{y^2}{36} = 1$$

Identify the transverse axis

Since the x^2 term is preceded by a plus sign, the transverse axis will be along the x-axis and the hyperbola would follow the equation of:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Find the foci

Since the transverse axis is along the x-axis the foci would be in the form of (-c, 0) and (c, 0)

$c^2 = a^2 + b^2$
$c^2 = 16 + 36$
$c^2 = 52$

$$c = \sqrt{52}$$

$$c = 2\sqrt{13}$$

The foci (-c, 0) and (c, 0) are ($-2\sqrt{13}$, 0) and ($2\sqrt{13}$, 0)

Exercise 3 (Continued):

Find the equations of the asymptotes

$b^2 = 36$
$b = 6$

$$\begin{aligned} y &= \pm\frac{b}{a}x \\ &= \pm\frac{6}{4}x \\ &= \pm\frac{3}{2}x \end{aligned}$$

Exercise 4: Convert the given equation into the standard form of a hyperbola by completing the square of x and y. Find the vertices, foci, and equations of the asymptotes. Sketch the graph.

$9y^2 - x^2 + 54y + 4x + 68 = 0$

Rewrite the equation grouping the x-terms and y-terms on the left and the constant on the right

$(9y^2 + 54y) + (-x^2 + 4x) = -68$

Factor so that the coefficients of the x^2 and y^2 terms is 1

$9(y^2 + 6y) - (x^2 - 4x) = -68$

Complete the square and simplify the equation

$$9\left(y^2+6y+\left(\frac{6}{2}\right)^2\right)-\left(x^2-4x+\left(\frac{-4}{2}\right)^2\right)=-68+9\left(\frac{6}{2}\right)^2-1\left(\frac{-4}{2}\right)^2$$

$$9\left(y^2+6y+(3)^2\right)-\left(x^2-4x+(-2)^2\right)=-68+9(3)^2-1(-2)^2$$

$$9(y+3)^2-(x-2)^2=-68+81-4$$

$$9(y+3)^2-(x-2)^2=9$$

$$\frac{9(y+3)^2}{9}-\frac{(x-2)^2}{9}=\frac{9}{9}$$

$$\frac{(y+3)^2}{1}-\frac{(x-2)^2}{9}=1$$

Exercise 4 (Continued):

Identify the transverse axis

Since the y^2 term is preceded by a plus sign, the transverse axis will be along the y-axis.

Find the center

$$\frac{(y-(-3))^2}{1}-\frac{(x-2)^2}{9}=1$$

(h, k) = (2, -3)

Find the vertices

$a^2 = 1$
a = 1

The vertices (h, k - a) and (h, k + a) are:

(2, -3 - 1) = (2, -4) and (2, -3 + 1) = (2, -2)

Find the foci

Since the transverse axis is along the y-axis the foci would be in the form of (h, k - c) and (h, k + c)

$c^2 = a^2 + b^2$
$c^2 = 1 + 9$
$c^2 = 10$
$c = \sqrt{10}$

The foci (h, k - c) and (h, k + c) are:

(2, -3 - $\sqrt{10}$) and (2, -3 + $\sqrt{10}$)

Find the equations of the asymptotes

$b^2 = 9$
b = 3

$$y = \pm\frac{a}{b}x$$
$$= \pm\frac{1}{3}x$$

Exercise 4 (Continued):

Since the center is at (2, -3), the asymptotes would be shifted 2 units to the right and 3 units down

$$y+3=\pm\frac{1}{3}(x-2)$$

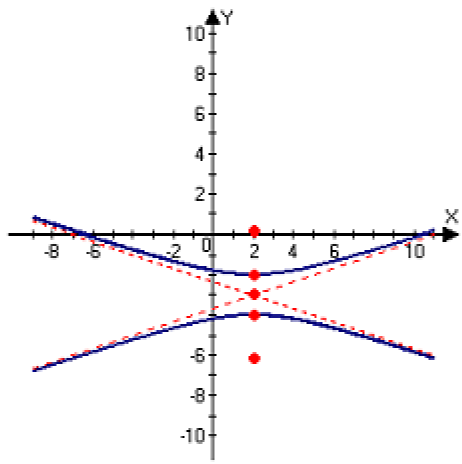
$$y+3=\frac{1}{3}(x-2) \qquad \text{or} \qquad y+3=-\frac{1}{3}(x-2)$$

$$y+3=\frac{1}{3}x-\frac{2}{3} \qquad\qquad y+3=-\frac{1}{3}x+\frac{2}{3}$$

$$y=\frac{1}{3}x-\frac{2}{3}-3 \qquad\qquad y=-\frac{1}{3}x+\frac{2}{3}-3$$

$$y=\frac{1}{3}x-\frac{11}{3} \qquad\qquad y=-\frac{1}{3}x-\frac{7}{3}$$

Sketch the graph

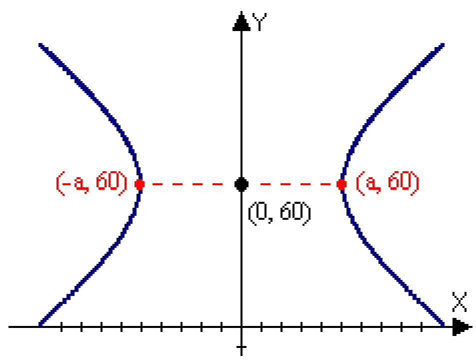


Exercise 5: The perpendicular cross section of a nuclear power station cooling tower form two branches of a hyperbola. Suppose the central cross section of the tower is modeled by the hyperbola $2500x^2 - 625(y - 60)^2 = 1{,}000{,}000$ (x and y are in feet). What is the minimum distance between the sides of the tower?

Rewrite the equation into standard form

$$2500x^2 - 625(y-60)^2 = 1{,}000{,}000$$
$$\frac{2500x^2}{1{,}000{,}000} - \frac{625(y-60)^2}{1{,}000{,}000} = \frac{1{,}000{,}000}{1{,}000{,}000}$$
$$\frac{x^2}{400} - \frac{(y-60)^2}{1{,}600} = 1$$

Draw a diagram of the problem



Find the value of a

$a^2 = 400$
$a = 20$

Find the minimum distance between the sides of the tower

The minimum distance would be the distance between the two vertices which would be twice the value of a

$2a = 2(20) = 40$

The minimum distance between the sides of the tower is 40 feet.