Review Exercise Set 23

Exercise 1: Find the focus and directrix of the parabola given by $y^2 = -12x$. Sketch the graph.

Exercise 2: Find the equation of a parabola in standard form with a focus at (0, 4) and a directrix at y = -4.

Exercise 3: Find the vertex, focus, and directrix of the given parabola. Sketch the graph.

$$(y + 3)^2 = -\frac{1}{2}(x + 5)$$

Exercise 4: Express the given parabola in standard form.

$3x^2 - 24x - 12y + 12 = 0$

Exercise 5: If a satellite dish with a parabolic surface is made with a diameter of 8 feet and a depth of 1 foot, then how far from the base of the dish should the receiver be placed?

Review Exercise Set 23 Answer Key

Exercise 1: Find the focus and directrix of the parabola given by $y^2 = -12x$. Sketch the graph.

Find the value of p

$y^2 = 4px$

$4p = -12$
$p = -3$

Find the focus

(p, 0)
(-3, 0)

Find the directrix

$x = -p$
$x = -(-3)$
$x = 3$

Exercise 2: Find the equation of a parabola in standard form with a focus at (0, 4) and a directrix at y = -4.

Find p

Focus is at (0, 4) so p = 4

Find the equation of the parabola

Since the focus is on the y-axis the equation will be in the form of:

$x^2 = 4py$
$x^2 = 4(4)y$
$x^2 = 16y$

Exercise 3: Find the vertex, focus, and directrix of the given parabola. Sketch the graph.

$(y + 3)^2 = -\frac{1}{2}(x + 5)$

Find the vertex by rewriting the equation into the form $(y - k)^2 = 4p(x - h)$

$(y - (-3))^2 = -\frac{1}{2}(x - (-5))$

The vertex (h, k) is at (-5, -3)

Find the value of p

$4p = -\frac{1}{2}$

$p = -\frac{1}{2} \times \frac{1}{4}$

$p = -\frac{1}{8}$

Find the focus

The focus is at (h + p, k)

$(h + p, k) = (-5 - \frac{1}{8}, -3)$

$(h + p, k) = (-\frac{41}{8}, -3)$

Find the directrix

The directrix is given by the equation x = h - p

$x = h - p$

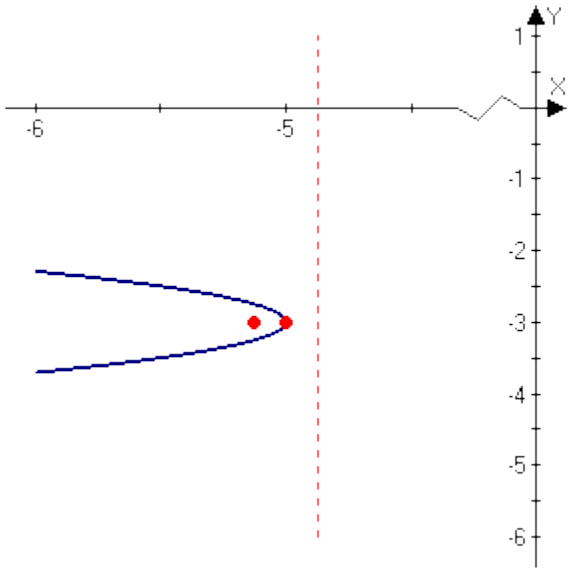
$x = -5 - (-\frac{1}{8})$

$x = -5 + \frac{1}{8}$

$x = \frac{39}{8}$

Exercise 3 (Continued):

Sketch the graph



Exercise 4: Express the given parabola in standard form.

$3x^2 - 24x - 12y + 12 = 0$

Isolate the terms involving x on the left side

$3x^2 - 24x = 12y - 12$

Factor the left side so that the x^2 term has a coefficient of 1

$3(x^2 - 8x) = 12y - 12$

Complete the square on the left side

$$3\left(x^2 - 8x + \left(\frac{-8}{2}\right)^2\right) = 12y - 12 + 3\left(\frac{-8}{2}\right)^2$$

$$3\left(x^2 - 8x + (-4)^2\right) = 12y - 12 + 3(-4)^2$$

$$3(x-4)^2 = 12y - 12 + 48$$

$$3(x-4)^2 = 12y + 36$$

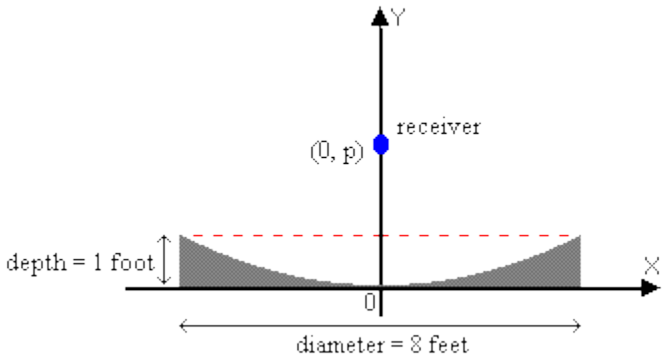
$$3(x-4)^2 = 12(y+3)$$

Exercise 5: If a satellite dish with a parabolic surface is made with a diameter of 8 feet and a depth of 1 foot, then how far from the base of the dish should the receiver be placed?

The receiver should be placed at the focus of the parabola so we need to find the value of p.

Draw a diagram of the problem

Let the dish be positioned so that its vertex is at the origin (0, 0)



Find the coordinates of one of the endpoints of dish

Since the dish is positioned at the origin the distance to the endpoints would be half of the dish's diameter.

$x = \frac{1}{2}$ (diameter of the dish)

$x = \frac{1}{2}$ (8)

$x = 4$

At the endpoints the value of y would be equal to the depth of the dish.

$y = 1$

Use the coordinates to find p

$x^2 = 4py$

$(4)^2 = 4p(1)$

$16 = 4p$

$4 = p$

The focus is at (0, 4).

Since the receiver should be placed at the focus, the receiver needs to be placed 4 feet from the base of the dish.