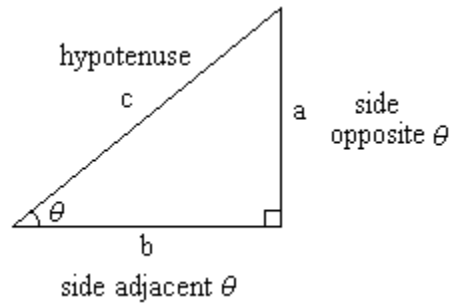


Applications of Trigonometric Functions

The previous sections have discussed the definitions of the basic trigonometric functions and how they relate to the right triangle. This information can then be applied to specific real world situations to solve for unknown values. First, let's quickly review the definition of the six trigonometric functions in terms of a right triangle.



$$\sin \theta = \frac{a}{c}$$

$$\csc \theta = \frac{c}{a}$$

$$\cos \theta = \frac{b}{c}$$

$$\sec \theta = \frac{c}{b}$$

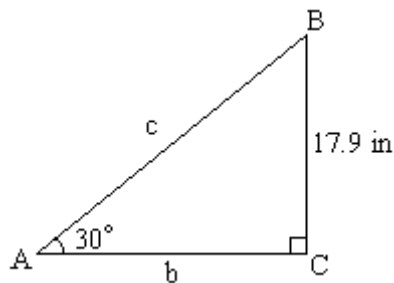
$$\tan \theta = \frac{a}{b}$$

$$\cot \theta = \frac{b}{a}$$

The trigonometric function that would be used will depend on the information that is known and what you are asked to find. There are two general situations when solving right triangles – (1) a side and an angle are known or (2) two sides are known. The following examples will cover both of these situations.

Solving a right triangle given one angle and one side

Example 1: Solve the right triangle shown below rounding all measurements to one decimal place (if necessary).



Example 1 (Continued):

Solution:

Identify the given information

$$\angle A = 30^\circ; \angle C = 90^\circ; \text{ and } a = 17.9 \text{ in}$$

Identify what needs to be found

$$\angle B; b; \text{ and } c$$

Find $\angle B$

The sum of the angles of a triangle must add up to equal 180 degrees. Since we know the measurements of angles A and C we can easily find the measurement of angle B.

$$\angle A + \angle B + \angle C = 180^\circ$$

$$30^\circ + \angle B + 90^\circ = 180^\circ$$

$$\angle B + 120^\circ = 180^\circ$$

$$\angle B = 60^\circ$$

Find b

Side b is adjacent to the angle that we were given and since we also know the length of side a we can use the tangent function to solve for b.

$$\tan A = \frac{a}{b}$$

$$\tan 30^\circ = \frac{17.9}{b}$$

$$b(\tan 30^\circ) = 17.9$$

$$b = \frac{17.9}{\tan 30^\circ}$$

$$b = 31 \text{ in}$$

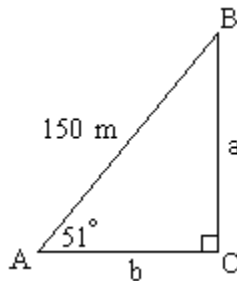
Find c

Side c is the hypotenuse of the triangle. The sine of angle A is equal to the length of the side opposite of angle A divided by the length of the hypotenuse. We know two of the values and can therefore easily solve for the unknown value using the sine function.

Example 1 (Continued):

$$\begin{aligned}\sin A &= \frac{a}{c} \\ \sin 30^\circ &= \frac{17.9}{c} \\ c(\sin 30^\circ) &= 17.9 \\ c &= \frac{17.9}{\sin 30^\circ} \\ c &= 35.8 \text{ in}\end{aligned}$$

Example 2: Solve the right triangle shown below rounding all measurements to one decimal place (if necessary).



Solution:

Identify the given information

$$\angle A = 51^\circ; \angle C = 90^\circ; \text{ and } c = 150 \text{ m}$$

Identify what needs to be found

$$\angle B; a; \text{ and } b$$

Find $\angle B$

$$\begin{aligned}\angle A + \angle B + \angle C &= 180^\circ \\ 51^\circ + \angle B + 90^\circ &= 180^\circ \\ \angle B + 141^\circ &= 180^\circ \\ \angle B &= 39^\circ\end{aligned}$$

Example 2 (Continued):

Find a

We know angle A and the hypotenuse, so the sine function can be used to determine the length of side a.

$$\sin A = \frac{a}{c}$$

$$\sin 51^\circ = \frac{a}{150}$$

$$150(\sin 51^\circ) = a$$

$$a \approx 116.6 \text{ m}$$

Find b

Side b is adjacent to angle A and since we know the measurements of angle A and the hypotenuse the cosine function can now be used to find b.

$$\cos A = \frac{b}{c}$$

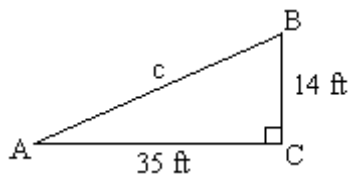
$$\cos 51^\circ = \frac{b}{150}$$

$$150(\cos 51^\circ) = b$$

$$b \approx 94.4 \text{ m}$$

Solving a right triangle given two sides

Example 3: Solve the right triangle shown below rounding all measurements to one decimal place (if necessary).



Example 3 (Continued):

Solution:

Identify the given information

$$\angle C = 90^\circ; a = 14 \text{ ft; and } b = 35 \text{ ft}$$

Identify what needs to be found

$$\angle A; \angle B; \text{ and } c$$

Find $\angle A$

Since we know the measurements of sides a and b we can use the tangent function to find the measurement of angle A.

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{14}{35}$$

$$A = \tan^{-1} \frac{14}{35}$$

$$A \approx 21.8^\circ$$

Find $\angle B$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$21.8^\circ + \angle B + 90^\circ \approx 180^\circ$$

$$\angle B + 111.8^\circ \approx 180^\circ$$

$$\angle B \approx 68.2^\circ$$

Find c

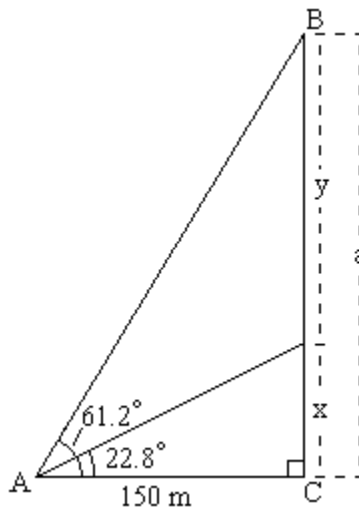
Since we know the lengths of the sides a and b the Pythagorean Theorem can be used to find the length of the hypotenuse. You do not want to use a trigonometric function involving either of the two angles we just found because they have been rounded off. Using the original information given and the Pythagorean Theorem will provide a much better estimate of the length of the hypotenuse.

Example 3 (Continued):

$$\begin{aligned}a^2 + b^2 &= c^2 \\(14)^2 + (35)^2 &= c^2 \\196 + 1225 &= c^2 \\1421 &= c^2 \\\sqrt{1421} &= c \\37.7 &\approx c\end{aligned}$$

There could also be a situation where you will have a combination of right triangles that are used to solve for an unknown amount.

Example 4: Solve the right triangles shown below for y rounding all measurements to one decimal place (if necessary).



Solution:

In order to find the value of y , we must first solve for a and x . We could then find y by subtracting x from a .

Find a

Side a is the full length of the side between B and C . Since the angle A and its adjacent side are known, we can use the tangent function to find a .

$$\begin{aligned}\tan A &= \frac{a}{b} \\\tan 61.2^\circ &= \frac{a}{150} \\150(\tan 61.2^\circ) &= a \\a &\approx 272.8 \text{ m}\end{aligned}$$

Example 4 (Continued):

Find x

Side x is also opposite of the angle A so we will once again use the tangent function to find the length of x. However, this time we will use the smaller angle of 22.8° for A.

$$\tan A = \frac{x}{b}$$

$$\tan 22.8^\circ = \frac{x}{150}$$

$$150(\tan 22.8^\circ) = x$$

$$x \approx 63.1 \text{ m}$$

Find y

$$x + y = a$$

$$63.1 + y \approx 272.8$$

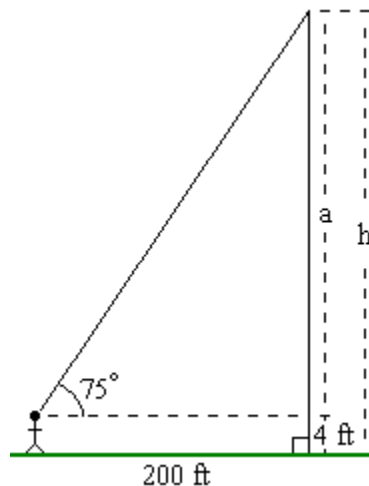
$$y \approx 209.7 \text{ m}$$

Now that you have an understanding of how the trigonometric functions are used to solve right triangles, let's look at some real world applications. The techniques we used in solving the previous examples can be applied in the areas of surveying and navigation.

Example 5: A child who is 4 feet tall is standing 200 feet away from the center of the base of the Tower of Americas. If the child must look up at an angle of 75° to see the top of the tower, how tall is the Tower of Americas?

Solution:

Draw diagram of the situation



Example 5 (Continued):

Find a

For the triangle involving a, we know both the measurements of angle A and the length of the side adjacent to it. Therefore, we can use the tangent function to determine a.

$$\tan A = \frac{a}{b}$$

$$\tan 75^\circ = \frac{a}{200}$$

$$200(\tan 75^\circ) = a$$

$$a \approx 746 \text{ ft}$$

Find h

The height of the Tower of Americas is equal to the value of a plus the height of the child.

$$h = 4 + a$$

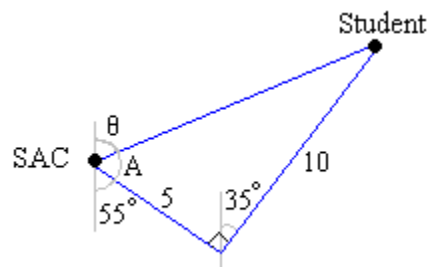
$$h \approx 4 + 746$$

$$h \approx 750 \text{ ft}$$

Example 6: A SAC student leaves the campus on a bearing of S 55° E and travels 5 miles before turning on a bearing of N 35° E. After traveling 10 miles on this new bearing what is the bearing of the student from SAC?

Solution:

Draw diagram of the situation



Identify known information

Right triangle

Adjacent side (b) = 5

Opposite side (a) = 10

Example 6 (Continued):

Identify what needs to be found

The angle A

The angle θ

Find angle A

Since the adjacent and opposite sides are known the tangent function is used to find the measurement of angle A

$$\tan A = \frac{a}{b}$$

$$\tan A = \frac{10}{5}$$

$$A = \tan^{-1} 2$$

$$A \approx 63.4^\circ$$

Find angle θ

Angle θ , A and the 55° angle form a straight line and are therefore supplementary angles whose sum must equal 180° .

$$\angle \theta + \angle A + 55^\circ = 180^\circ$$

$$\angle \theta + 63.4^\circ + 55^\circ \approx 180^\circ$$

$$\angle \theta + 118.4^\circ \approx 180^\circ$$

$$\angle \theta \approx 61.6^\circ$$

The student's bearing from SAC is N 61.6° E.

The cosine and sine functions are used in the area of simple harmonic motion because of how the oscillatory motion of an object will resemble the graphs of the trigonometric functions. The distance an object in simple harmonic motion travels can be expressed using either of the two trigonometric functions (cosine or sine).

$$d = a \cos \omega t \quad \text{or} \quad d = a \sin \omega t; \quad \text{period} = \frac{2\pi}{\omega}$$

where d is the object's distance from the origin (or its rest position) and the absolute value of a represents the amplitude or the maximum distance the object can travel from its rest position.

The determination of which function to use depends on where the object is located at the time $t = 0$. If the object is at its maximum distance from its rest position when $t = 0$ then the cosine function will be used. If the object is at its rest position when $t = 0$ then the sine function is used.

Example 7: A ball attached to a spring is pulled 5 inches below its rest position and then released. The period of the ball's motion is 4 seconds. Write the equation for the ball's simple harmonic motion.

Solution:

Identify given information

$$\begin{aligned}d &= -5 \text{ when } t = 0 \\ \text{period} &= 4\end{aligned}$$

Determine which trigonometric function will be used

Since the object will be at its maximum distance from its rest position when $t = 0$ the cosine function would be used to write the equation.

$$d = a \cos \omega t$$

Identify what information needs to be found

In order to write the equation we will need to find a and ω

Find a

$|a|$ is the ball's maximum displacement which occurs at $t = 0$.

$$a = -4$$

Find ω

The period is 4 seconds so we can find the value of ω with the equation

$$\text{period} = \frac{2\pi}{\omega}$$

$$4 = \frac{2\pi}{\omega}$$

$$4\omega = 2\pi$$

$$\omega = \frac{\pi}{2}$$

Example 7 (Continued):

Write the equation

$$d = a \cos \omega t$$

$$d = -4 \cos \frac{\pi}{2} t$$