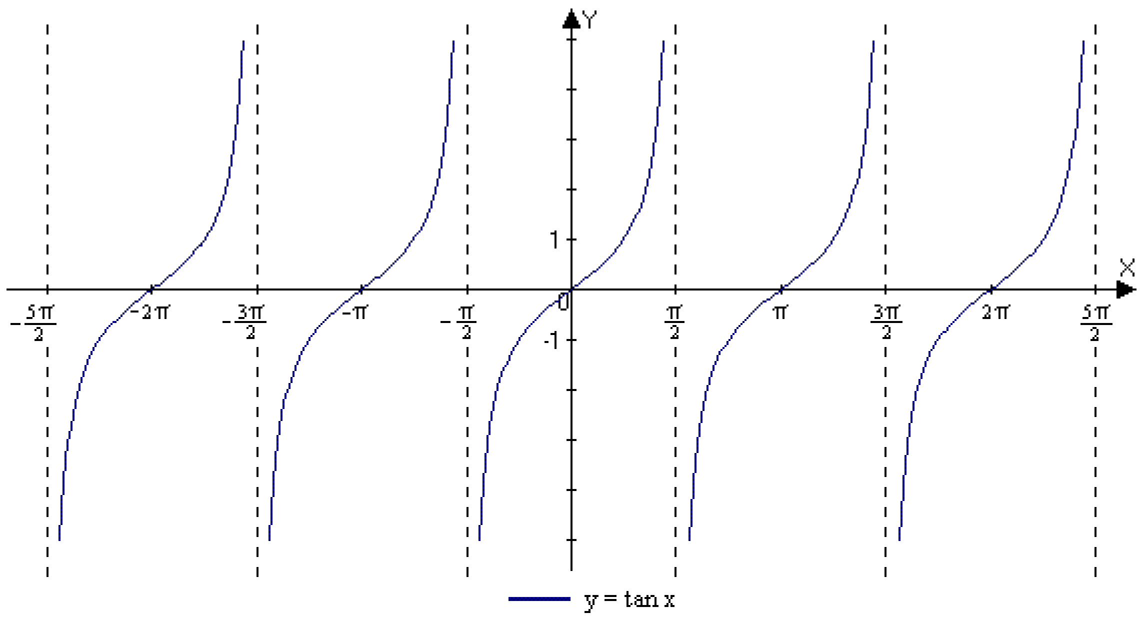
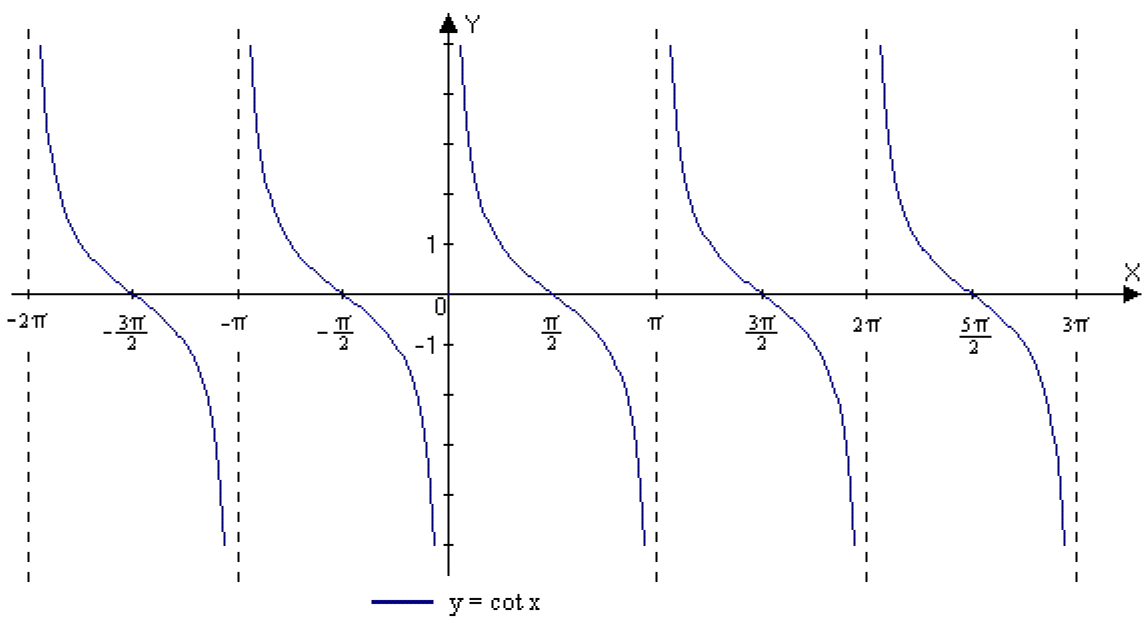
Graphs of Other Trigonometric Functions

Tangent and Cotangent

- In graphing y= A tan (Bx + C) and y= A cot (Bx + C), we are basically using the same procedures used in graphing sine and cosine.

- The graphs for basic tangent and cotangent functions:





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Drawing the Graph

- To sketch a tangent and cotangent graph one needs to know how the constants *A*, *B*, and *C* of y = *A* tan (*B*x + *C*) graph, affect the regular y = tan x and y = cot x graphs.
  - First off, the amplitude is not an accurate factor for the tangent and cotangent functions because they both depart from the x-axis to infinity on both ends.
  - Second, *A* affects the graph by either making it steeper or less steeper. If $|A| > 1$, then the graph is steeper. If $|A| < 1$, then the graphs is less steep.
  - Third, If *A* is a negative number, the graph is a reflection across the x-axis.
  - The constants *B* and *C* have the same affect on the graph like in sine and cosine, change in period (*B*), and phase shift (*C*).

- Tangent and cotangent both have the same period of π, therefore each complete one cycle as the Bx + C goes from 0 → π.
  - In other words, if you are solving for x, then x varies from

$$x = -C/B \quad \rightarrow \quad x = -C/B + \pi/B$$

- y = *A* tan (*B*x + *C*) and y = *A* cot (*B*x + *C*) have a period of π/*B* and a phase shift of –*C*/*B*.

- The general graph is shifted to the right if –*C*/*B* is positive, and to the left if –*C*/*B* is negative.

Graphing y = A cot (Bx + C) – Without Phase Shift

1st → We find the period and phase shift for y = 2 cot (2x).

- Solve for x:

Phase Shift → $Bx + C = 0$
$2x + 0 = 0$
$2x/2 = 0/2$
$x = 0$ **Phase shift = 0**

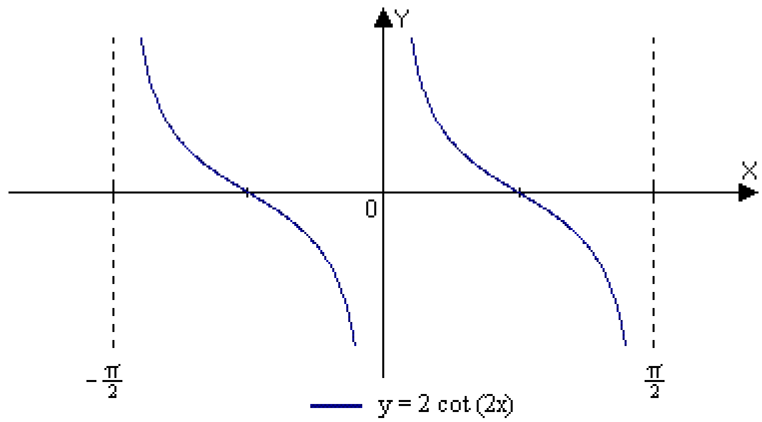
Period → $Bx + C = \pi$
$2x + 0 = \pi$
$2x/2 = \pi/2$
$x = \pi/2$ **Period = π/2**

(*C* = 0, therefore there is no phase shift)

Graphing y = A cot (Bx + C) – Without Phase Shift Continued …

2nd → Then, we sketch the graph within the intervals $-\pi/2 < x < \pi/2$.

- As 2x varies from 0 to π, y = 2 cot (2x) completes one cycle.
  - Graph:



Graphing y = A cot (Bx + C) – With Phase Shift

- Let's find the period and phase shift for y = cot (πx/2 + π/4)
- Solve for x:

Phase Shift →

Bx + C = 0
πx/2 + π/4 = 0
πx/2 = -π/4
2/π(πx/2) = (-π/4) (2/π) (multiply the reciprocal of π/2)
x= -1/2

Phase shift = -1/2

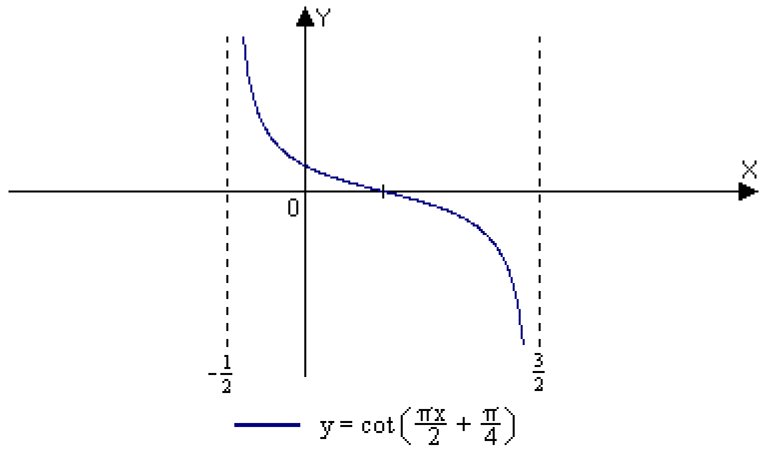
Period →

Bx + C = π
πx/2 + π/4 = π
πx/2 = -π/4 + π
2/π(πx/2) = (-π/4 + π) (2/π) (multiply the reciprocal of π/2)
x = 2

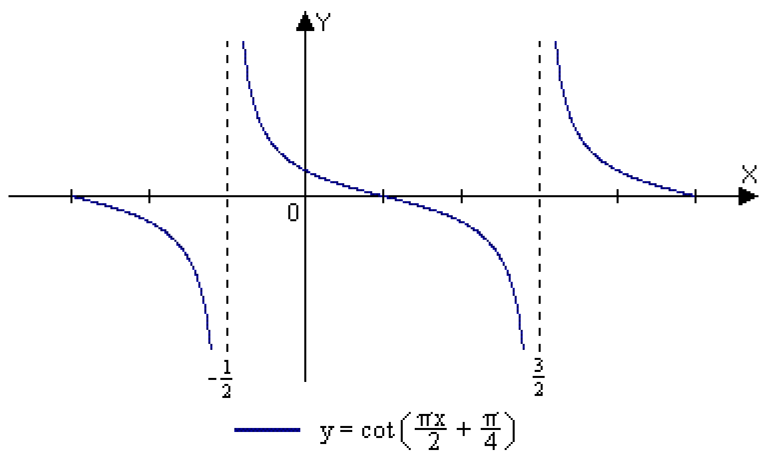
Period = 2π

Graphing y = A cot (Bx + C) – With Phase Shift Continued …

Sketch the graph (only one period) starting at x = -1/2 (the phase shift), and ending at x = -1/2 + 2 (the phase shift plus the period) which will be x = 3/2.



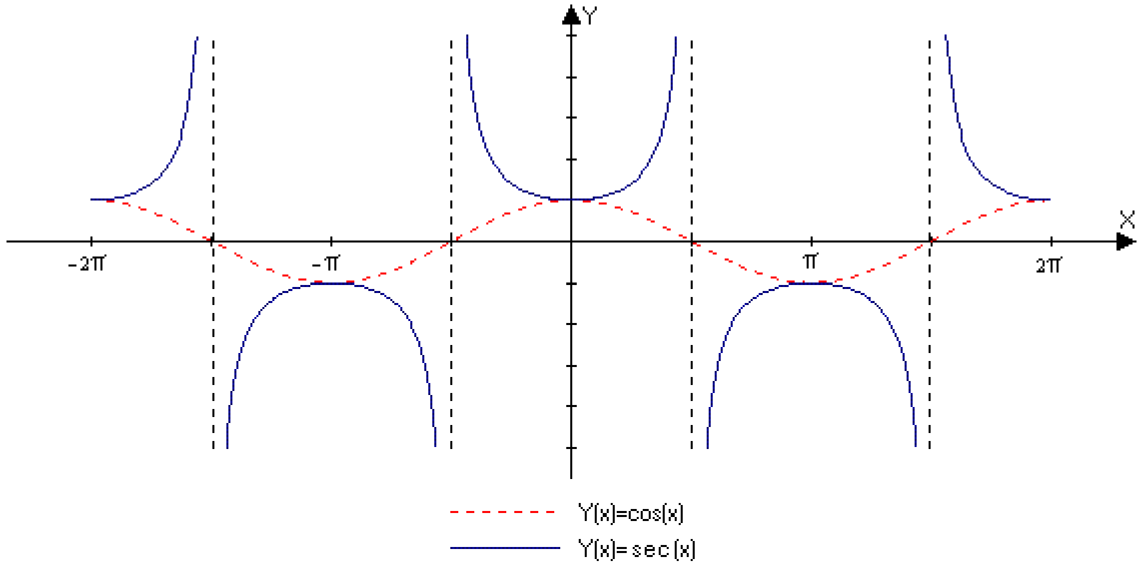
- Extend the graph of y = cot (πx/2 + π/4) over the interval (-3/2, 2 ½)



Secant and Cosecant

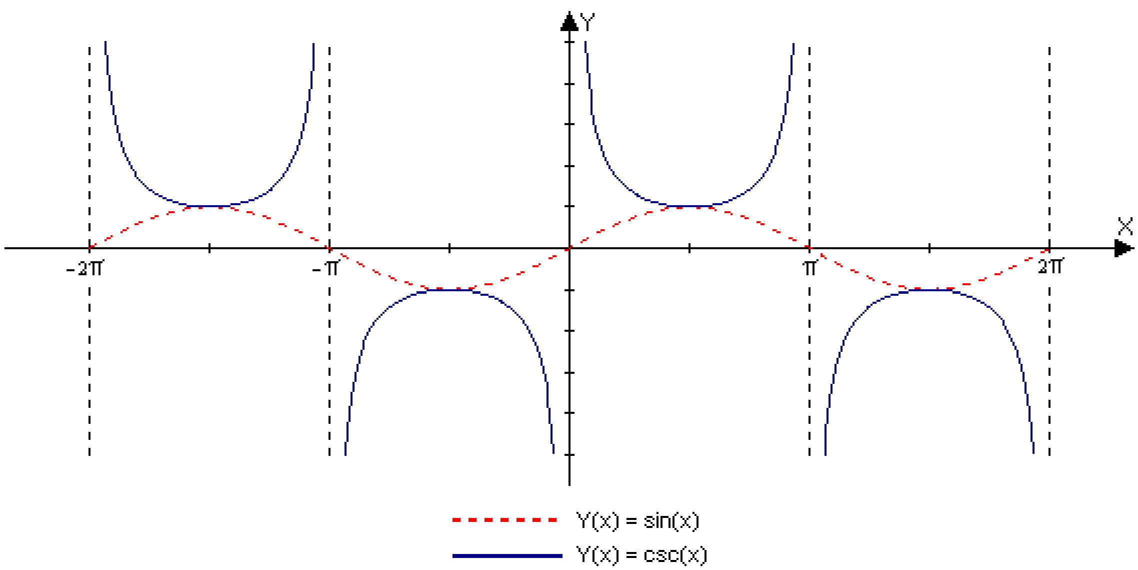
y = sec x
- Period = 2π
- Symmetric with respect to the y-axis.
- Domain = all real numbers; x does not equal to π/2 + kπ, k an integer.
- Range = all real numbers; y ≤ -1 or y ≥ 1
- Discontinuous at x = π/2 + kπ, k an integer.

Secant and Cosecant Continued …



y = csc x

– Period = 2π
– Symmetric with respect to the origin.
– Domain = all real numbers; x does not equal to kπ, k an integer.
– Range = all real numbers; y ≤ -1 or y ≥ 1
– Discontinuous at x = kπ, k an integer.



Graphing Secant and Cosecant

- Like the tangent and cotangent functions, amplitude does not play an important role for secant and cosecant functions.

- Both have the same period of 2π, so we solve the phase shift and period with $Bx + C = 0$ & $Bx + C = 2\pi$

- It is easier to graph $y = A \sec(Bx + C)$ or $y = A \csc(Bx + C)$ by graphing $y = (1/A) \cos(Bx + C)$ or $y = (1/A) \sin(Bx + C)$, the cosine and sine graph with dashed curve, then taking the reciprocal of the sine and cosine graph.

Secant Example

- $Y = \sec(2x + \pi)$

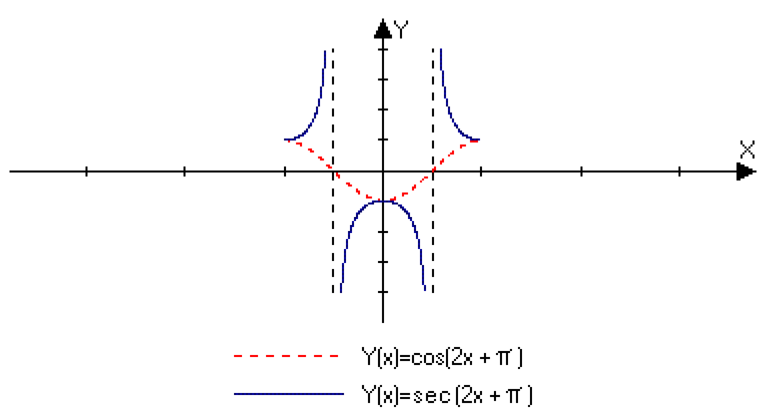
  – Find the period and phase shift first:

Phase shift → $Bx + C = 0$
$2x + \pi = 0$
$2x = -\pi$
$2x/2 = -\pi/2$ **Phase shift: $x = -\pi/2$**

Period → $Bx + C = 2\pi$
$2x + \pi = 2\pi$
$2x = -\pi + 2\pi$
$2x/2 = (-\pi + 2\pi)/2$ **Period: $x = \pi$**

- Because $\sec(2x + \pi) = 1/\cos(2x + \pi)$, we can graph $y = \cos(2x + \pi)$, starting from $-\pi/2$ to $-\pi/2 + \pi$, which is one cycle.

- Then we take the reciprocals of the graph.



Secant Example Continued…

- The vertical asymptotes go through the x-intercepts of the cosine graph to direct the sketching of the secant function.
- Extend graph to its interval ($-3\pi/4$, $3\pi/4$)