Review Exercise Set 4

Exercise 1: If the point (-3, 7) is a point on the terminal side of angle θ then find the exact value of each of the six trigonometric functions of θ.

Exercise 2: Using the given conditions, find the exact value of each of the remaining trigonometric functions of θ.

$$\cos\theta = \frac{3}{7} \ \ and \ \ \sin\theta < 0$$

Exercise 3: Using the given conditions, find the exact value of each of the remaining trigonometric functions of θ.

$$\cot\theta = 3 \ \ and \ \ \cos\theta < 0$$

Exercise 4: Use reference angles to find the value of the given expression.

$\cos 495°$

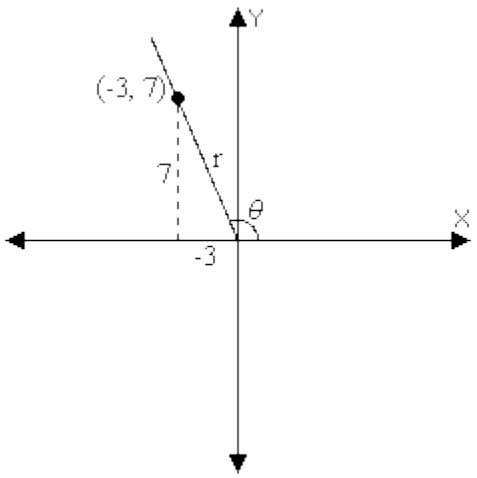
Exercise 5: Find the exact value of the given expression. Write the answer as a single fraction.

$$\left(\cos\frac{5\pi}{3}\right)\left(\cot\frac{5\pi}{6}\right)+\sin\frac{3\pi}{4}$$

Review Exercise Set 4 Answer Key

Exercise 1: If the point (-3, 7) is a point on the terminal side of angle θ then find the exact value of each of the six trigonometric functions of θ.

Graph the point to get a visual picture of the problem



Find the length of the hypotenuse "r"

$$\begin{aligned} r^2 &= x^2 + y^2 \\ &= (-3)^2 + (7)^2 \\ &= 9 + 49 \\ &= 58 \\ r &= \sqrt{58} \end{aligned}$$

Find the value of the trig functions

$$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{58}} = \frac{7\sqrt{58}}{58}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{58}} = -\frac{3\sqrt{58}}{58}$$

$$\tan\theta = \frac{y}{x} = \frac{7}{-3} = -\frac{7}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{58}}{7}$$

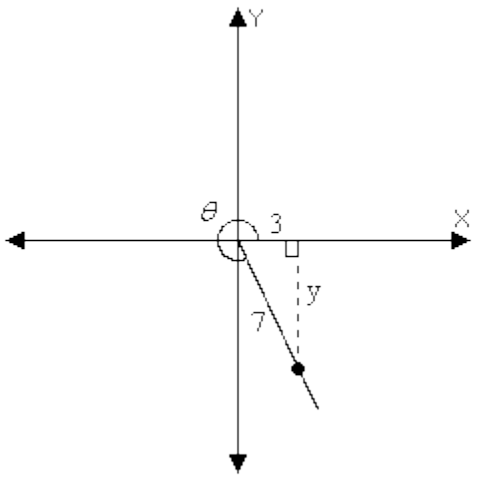
$$\sec\theta = \frac{r}{x} = \frac{\sqrt{58}}{-3} = -\frac{\sqrt{58}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{7} = -\frac{3}{7}$$

Exercise 2: Using the given conditions, find the exact value of each of the remaining trigonometric functions of θ.

$$\cos\theta = \frac{3}{7} \ \ and \ \ \sin\theta < 0$$

Graph the angle θ based on the given conditions to get a visual picture of the problem



Find the length of the side "y"

$$\begin{aligned} r^2 &= x^2 + y^2 \\ (7)^2 &= (3)^2 + y^2 \\ 49 &= 9 + y^2 \\ 40 &= y^2 \\ -\sqrt{40} &= y \\ -2\sqrt{10} &= y \end{aligned}$$

Remember since the point is in the fourth quadrant y must be negative.

Find the value of the remaining trig functions

$$\sin\theta = \frac{y}{r} = \frac{-2\sqrt{10}}{7} = -\frac{2\sqrt{10}}{7} \qquad \cos\theta = \frac{x}{r} = \frac{3}{7} \qquad \tan\theta = \frac{y}{x} = \frac{-2\sqrt{10}}{3} = -\frac{2\sqrt{10}}{3}$$

Exercise 2 (Continued):

$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

$$= \frac{7}{-2\sqrt{10}} \qquad = \frac{7}{3} \qquad = \frac{-3}{-2\sqrt{10}}$$

$$= -\frac{7\sqrt{10}}{20} \qquad\qquad = -\frac{3\sqrt{10}}{20}$$

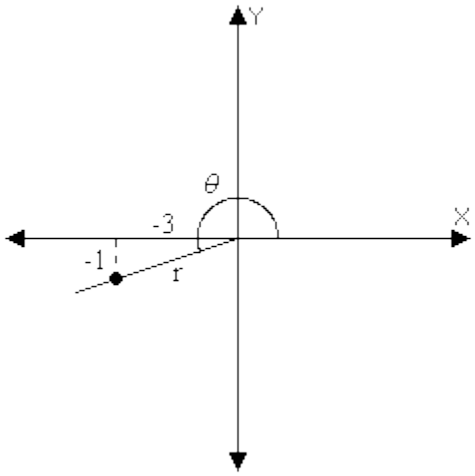
Exercise 3: Using the given conditions, find the exact value of each of the remaining trigonometric functions of θ.

$$\cot\theta = 3 \;\; and \;\; \cos\theta < 0$$

Graph the angle θ based on the given conditions to get a visual picture of the problem

Since cosine is negative and cotangent is positive x and y must both be negative which would put the angle in the third quadrant.

$$\cot\theta = 3 = \frac{-3}{-1} = \frac{x}{y}$$



Find the length of the hypotenuse "r"

$$r^2 = x^2 + y^2$$
$$= (-3)^2 + (-1)^2$$
$$= 9 + 1$$
$$= 10$$
$$r = \sqrt{10}$$

Exercise 3 (Continued):

Find the value of the remaining trig functions

$$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10} \qquad \cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10} \qquad \tan\theta = \frac{y}{x} = \frac{-1}{-3} = \frac{1}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{10}}{-1} = -\sqrt{10} \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{-3} = -\frac{\sqrt{10}}{3} \qquad \cot\theta = \frac{x}{y} = \frac{-3}{-1} = 3$$

Exercise 4: Use reference angles to find the value of the given expression.

cos 495°

Find a positive coterminal angle that is less than 360°

495° - 360° = 135°

Determine the quadrant where this coterminal angle is located

135° is between 90° and 180° so it is located in quadrant II

Find the reference angle

Since the coterminal angle is in quadrant II we will subtract it from 180° to find the reference angle.

180° - 135° = 45°

Find the value of the expression using the reference angle

Cosine is negative in quadrant II so we would place a negative sign in front of the function

cos 495° = -cos 45°

Exercise 4 (Continued):

$$\cos 495^\circ = -\frac{1}{\sqrt{2}}$$
$$= -\frac{\sqrt{2}}{2}$$

Exercise 5: Find the exact value of the given expression. Write the answer as a single fraction.

$$\left(\cos\frac{5\pi}{3}\right)\left(\cot\frac{5\pi}{6}\right)+\sin\frac{3\pi}{4}$$

$$\left(\cos\frac{5\pi}{3}\right)\left(\cot\frac{5\pi}{6}\right)+\sin\frac{3\pi}{4} = \left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}\right)+\frac{\sqrt{2}}{2}$$
$$= \frac{3}{2}+\frac{\sqrt{2}}{2}$$
$$= \frac{3+\sqrt{2}}{2}$$