Review Exercise Set 7

Exercise 1: Find the exact value of the given trigonometric expression.

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Exercise 2: Use a calculator to find the value of the given trigonometric expression rounded to two decimal places.

$$\cos^{-1}(0.319)$$

Exercise 3: Find the exact value of the given trigonometric expression (if possible).

$$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$$

Exercise 4: Find the exact value of the given trigonometric expression (if possible).

$$\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$$

Exercise 5: Use a right triangle to write the given trigonometric expression as an algebraic expression. (Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x)

$$\sin\left[\cos^{-1}\left(\frac{5}{x}\right)\right]$$

Review Exercise Set 7 Answer Key

Exercise 1: Find the exact value of the given trigonometric expression.

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\sin\theta = \frac{\sqrt{3}}{2}; \quad where\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\theta = \frac{\pi}{3}$$

Exercise 2: Use a calculator to find the value of the given trigonometric expression rounded to two decimal places.

$$\cos^{-1}(0.319)$$

$$\theta = \cos^{-1}(0.319)$$

$$\cos\theta = 0.319; \quad where\ \ 0 \le \theta \le \pi$$

$$\theta \approx 1.25\ radians$$

Exercise 3: Find the exact value of the given trigonometric expression (if possible).

$$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$$

Solve the inner most expression first

$$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$\sin\theta = -\frac{1}{2}; \quad where\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\theta = -\frac{\pi}{6}$$

Exercise 3 (Continued):

Substitute the determined value of theta into the expression

$$\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \tan(\theta)$$
$$= \tan\left(-\frac{\pi}{6}\right)$$

Evaluate the expression

Since the angle is in quadrant IV the tangent must be negative. So we can use the reference angle of $\frac{\pi}{6}$ and place a negative sign in front of the tangent function.

$$\tan\left(-\frac{\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right)$$
$$= -\frac{1}{\sqrt{3}}$$
$$= -\frac{\sqrt{3}}{3}$$

Exercise 4: Find the exact value of the given trigonometric expression (if possible).

$$\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$$

Solve the inner most expression first

$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$
$$\tan\theta = \frac{3}{4}; \quad where\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\tan\theta = \frac{y}{x}$$

so x = 4 and y = 3

Exercise 4 (Continued):

$$r^2 = x^2 + y^2$$
$$r^2 = (4)^2 + (3)^2$$
$$r^2 = 25$$
$$r = 5$$

Use the values of x, y, and r to evaluate the expression

$$\cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right] = \cos(\theta)$$
$$= \frac{x}{r}$$
$$= \frac{4}{5}$$

Exercise 5: Use a right triangle to write the given trigonometric expression as an algebraic expression. (Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x)

$$\sin\left[\cos^{-1}\left(\frac{5}{x}\right)\right]$$

Use the inner most expression to solve for the opposite side

$$\theta = \cos^{-1}\left(\frac{5}{x}\right)$$
$$\cos\theta = \frac{5}{x}; \quad where \;\; 0 \le \theta \le \pi$$

Given that x is positive the angle θ would be in quadrant I which would make the sine function positive as well.

$$\cos\theta = \frac{x}{r}$$

so x = 5 and r = x

$$r^2 = x^2 + y^2$$
$$x^2 = (5)^2 + y^2$$
$$x^2 - 25 = y^2$$
$$\sqrt{x^2 - 25} = y$$

Exercise 5 (Continued):

Use the values of x, y, and r to evaluate the expression

$$\sin\left[\cos^{-1}\left(\frac{5}{x}\right)\right] = \sin(\theta)$$

$$= \frac{y}{r}$$

$$= \frac{\sqrt{x^2 - 25}}{x}$$