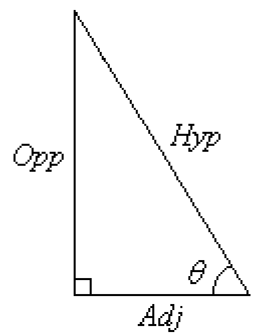
Right Triangle Trigonometry

In this section, we are interested in using trigonometric functions to compute the values of all angles and sides of any right triangle, given the value of any two sides of the triangle or the value of one side and one acute angle. This process is called "solving the right triangle."

Consider the right triangle shown below, where angle θ is one of the acute angles. We may define the functions in terms of the hypotenuse (Hyp), the side adjacent (Adj) to angle θ, and the side opposite (Opp) the angle θ for any right triangle, as shown below.



$$\sin\theta = \frac{Opp}{Hyp} \qquad \csc\theta = \frac{Hyp}{Opp}$$

$$\cos\theta = \frac{Adj}{Hyp} \qquad \sec\theta = \frac{Hyp}{Adj}$$

$$\tan\theta = \frac{Opp}{Adj} \qquad \cot\theta = \frac{Adj}{Opp}$$

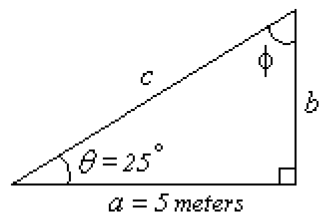
In order to solve right triangles given the lengths of two sides or one side and the value of one acute angle, it will be necessary to compute the value of a trigonometric function of one of the angles. We can use a calculator to do this.

We may also need a calculator to compute the value of an inverse trigonometric function, that is, the value of an angle given the value of a trigonometric function of that angle. The inverse function of a sine function, for example, can be written as $\sin^{-1}$ or as arcsin.

To illustrate, we may remember that $\sin(60°) = \frac{\sqrt{3}}{2}$. So for the inverse function, we may write

$\sin^{-1}\frac{\sqrt{3}}{2} = 60°$, or $\arcsin\frac{\sqrt{3}}{2} = 60°$.

Example 1: Solve the right triangle shown below with the length of side a = 5 meters and angle $\theta = 25°$.



Solution:

We are given the value of one of the angles, so we can find the value of the other acute angle of the right triangle by subtracting from 90 degrees.

angle $\phi = 90 - \theta = 90 - 25 = 65°$

Now we can use a trigonometric function of one of the angles to compute the length of one of the unknown sides. (Use a calculator to find the value of the trigonometric function for a given angle.)

$$\begin{aligned} \cos\theta &= \cos(25°) \\ &= 0.9063 \\ &= \frac{a}{c} \\ &= \frac{5}{c} \end{aligned}$$

$$c = \frac{5}{.9063} = 5.5169 \ \ meters$$

To compute the length of the other unknown side, we can use another trigonometric function of the angle.

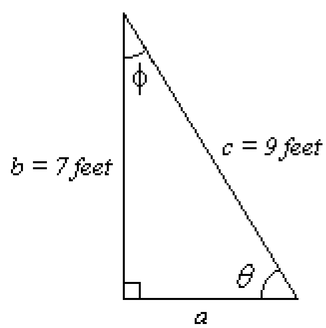
$$\begin{aligned} \tan\theta &= \tan(25°) \\ &= 0.4663 \\ &= \frac{b}{a} \\ &= \frac{b}{5} \end{aligned}$$

$$b = 5(.4663) = 2.3315 \ \ meters$$

Alternately, we could have used the Pythagorean Theorem to solve for side *b* after having found side *c*.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ b^2 &= c^2 - a^2 \\ b &= \sqrt{c^2 - a^2} \\ &= \sqrt{5.5169^2 - 5^2} \\ &= \sqrt{30.4362 - 25} \\ &= \sqrt{5.4362} \\ &= 2.3315 \ \ \textit{meters} \end{aligned}$$

Example 2: Solve the right triangle shown below with the length of side $b = 7$ feet and side $c = 9$ feet.



Solution:

First we can use a trigonometric function to solve for angle θ. We know that

$$\sin\theta = \frac{b}{c} = \frac{7}{9} = .7778$$

If we now use the inverse sine function, we can find angle θ.

$$\theta = \sin^{-1} .7778 = 51.06° \qquad \text{(using a calculator)}$$

To find angle ϕ in the right triangle, we can subtract angle θ from 90 degrees.

$$\phi = 90 - \theta = 90 - 51.06 = 38.94°$$

Example 2 (Continued):

Finally, we need to solve for side *a*.

$$\cos\theta = \cos 51.06° = .6285 = \frac{a}{c} = \frac{a}{9}$$

$$a = 9(.6285) = 5.6565 \quad feet$$

Alternatively, we could have used the Pythagorean Theorem to solve for side *a*.