Review Exercise Set 17

Exercise 1: Solve the given expression for x.

$$\left(\frac{2}{3}\right)^{x+1} = \frac{27}{8}$$

Exercise 2: Solve the given expression exactly for x. Do not approximate the answer.

$$\ln(5x) = 3$$

Exercise 3: Solve the given expression for x. Round answer to three decimal places.

$$e^{2x} + 7e^{x} - 3 = 0$$

Exercise 4: Solve the given expression for x. Round answer to three decimal places.

$$\log_3 (1 - x) - \log_3 (x + 2) = \log_3 x$$

Exercise 5: Solve the given expression for x. Round answer to three decimal places.

$$\ln(2 - x) - \ln(x + 1) = \ln(x + 3) - \ln(x)$$

Review Exercise Set 17 Answer Key

Exercise 1: Solve the given expression for x.

$$\left(\frac{2}{3}\right)^{x+1} = \frac{27}{8}$$

Rewrite the right-hand side so that it has the same base as the left-hand side

$$\left(\frac{2}{3}\right)^{x+1} = \frac{3^3}{2^3}$$
$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{3}{2}\right)^{3}$$
$$\left(\frac{2}{3}\right)^{x+1} = \left(\frac{2}{3}\right)^{-3}$$

Use the properties of exponents to set the two exponents equal to each other and solve for x

$x + 1 = -3$
$x = -4$

Exercise 2: Solve the given expression exactly for x. Do not approximate the answer.

$\ln(5x) = 3$

Convert into exponential form and solve for x

$e^3 = 5x$
$\frac{1}{5}e^3 = x$

Exercise 3: Solve the given expression for x. Round answer to three decimal places.

$e^{2x} + 7e^x - 3 = 0$

Use the quadratic equation

$(e^x)^2 + 7e^x - 3 = 0$

$a = 1;\ b = 7;\ c = -3$

Exercise 3 (Continued):

$$e^x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$e^x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-3)}}{2(1)}$$
$$e^x = \frac{-7 \pm \sqrt{49+12}}{2}$$
$$e^x = \frac{-7 \pm \sqrt{61}}{2}$$

$$e^x = \frac{-7-\sqrt{61}}{2} \quad or \quad e^x = \frac{-7+\sqrt{61}}{2}$$
$$\ln e^x = \ln\frac{-7-\sqrt{61}}{2} \qquad \ln e^x = \ln\frac{-7+\sqrt{61}}{2}$$
$$x = \ln\frac{-7-\sqrt{61}}{2} \qquad \ln e^x = \ln\frac{-7+\sqrt{61}}{2}$$
$$no\ solution \qquad x \approx -0.904$$

It is not possible to take the natural log of a negative number so $x = \ln\frac{-7-\sqrt{61}}{2}$ is not a possible solution. The only answer is that x is approximately -0.904.

Exercise 4: Solve the given expression for x. Round answer to three decimal places.

$\log_3 (1 - x) - \log_3 (x + 2) = \log_3 x$

Use the quotient rule of logarithms to combine the logs on the left-hand side together into a single log

$$\log_3\left(\frac{1-x}{x+2}\right) = \log_3 x$$

Use the one-to-one property of logs to rewrite the equation without the logs

$$\frac{1-x}{x+2} = x$$

Exercise 4 (Continued):

Solve for x

1 - x = x(x + 2)
1 - x = x^2 + 2x
0 = x^2 + 3x - 1

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{-3 \pm \sqrt{9+4}}{2} \\ &= \frac{-3 \pm \sqrt{13}}{2} \end{aligned}$$

$$x = \frac{-3+\sqrt{13}}{2} \quad or \quad x = \frac{-3-\sqrt{13}}{2}$$

$$x \approx 0.303 \qquad no\ solution$$

It is not possible to take the log of a negative number so $x = \frac{-3-\sqrt{13}}{2}$ is not a possible solution. The only answer is that $x = \frac{-3+\sqrt{13}}{2}$ or approximately 0.303.

Exercise 5: Solve the given expression for x. Round answer to three decimal places.

ln(2 - x) - ln(x + 1) = ln(x + 3) - ln(x)

Use the quotient rule of logarithms to combine the logs on both sides of the equation

$$\ln\left(\frac{2-x}{x+1}\right) = \ln\left(\frac{x+3}{x}\right)$$

Use the one-to-one property of logs to rewrite the equation without the logs

$$\frac{2-x}{x+1} = \frac{x+3}{x}$$

Exercise 5 (Continued):

Solve for x

x(2 - x) = (x + 3)(x + 1)
2x - x^2 = x^2 + 4x + 3
0 = 2x^2 + 2x + 3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{-2 \pm \sqrt{4 - 24}}{4}$$

$$= \frac{-2 \pm \sqrt{-20}}{4}$$

$$= \frac{-2 \pm 2i\sqrt{5}}{4}$$

This equation has no real solutions.