Average Rate of Change

In previous sections, we looked at some examples that dealt with travel time in a car. This type of example is very familiar to most of us, because we have all driven or been a passenger in a car. As seen in the previous section, when you drive at a faster rate, you will arrive at your destination earlier. In this section, we will look at the method of determining that rate. This method can also be used for many other situations, which deal with average rates of change, such as population growth, merchandise sales, etc.

Average Rate of Change:

It is very simple to calculate the rate or speed at which you are traveling; therefore, we will use this situation to develop the formula used in calculating this value.

Two friends, Holly and Carolyn are running in the "Race for the Cure." This is a 5-kilometer race. If they finish the race in 30 minutes, then we know that they were running at a rate of 10 km/h. This simple calculation can be represented by the following equation where the speed *(s)* is equal to the distance *(d)* traveled, divided by the time *(t)* it took to travel that distance.

$$s = \frac{d}{t}$$

$$s = \frac{5}{0.5}$$

$$s = 10$$

So they were traveling at a speed of 10km/h. (Notice, we took the time to be 0.5h, because they ran the race in 30 minutes.)

By rearranging the equation, we have the distance formula $d = s \cdot t$. We can write this equation in function notation where d is a function of time.

$$d(t) = s \cdot t$$

Now, what we are about to do may seem more complicated than necessary, but it will allow us to develop a method which will be very useful with many types of problems. Let's think about how we calculated the value of 10km/h initially. Without consciously thinking about it, we just subtracted the beginning distance (in this case, zero) from the ending distance (5km). Then we divided this value by the total time elapsed, which is the ending time (0.5h) minus the beginning time (0h).
If we put this in equation form, we have

$$s = \frac{5-0}{0.5-0} = \frac{5}{0.5} = 10$$

Using the function notation stated above, we have

$$s = \frac{d(0.5) - d(0)}{0.5-0}$$

This equation can be generalized to give the following:

Def: Average Rate of Change:

The *average rate of change* of the function $y = f(x)$ between $x = a$ and $x = b$ is

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$

We'll begin with an example, which will allow us to become familiar with the formula.

Example 1:

For the function $f(x) = 2x^2 - 1$, find the average rate of change of the function between the following points:

(a) $x = 2$ and $x = 1$
(b) $x = 0$ and $x = 3$

Solution:

(a) Average rate of change $= \dfrac{f(2) - f(1)}{2-1}$

$$= \frac{\left[2(2)^2 - 1\right] - \left[2(1)^2 - 1\right]}{2-1}$$

$$= \frac{[8-1]-[2-1]}{2-1}$$

$$= \frac{[7]-[1]}{2-1} = \frac{6}{1} = 6$$

Example 1 (Continued):

(b) Average rate of change $= \dfrac{f(0)-f(3)}{0-3}$

$$= \frac{\left[2(0)^2-1\right]-\left[2(3)^2-1\right]}{0-3}$$

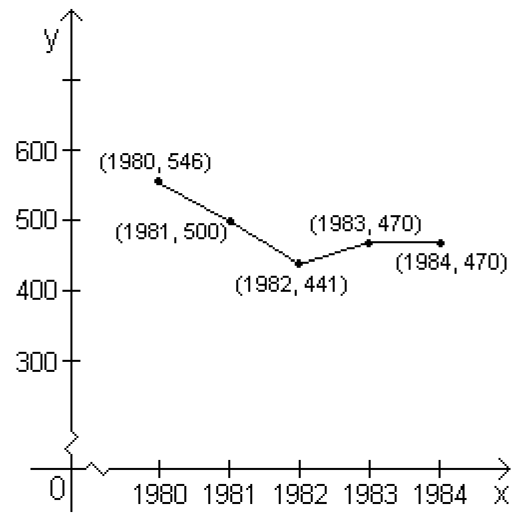
$$= \frac{[0-1]-[18-1]}{0-3}$$

$$= \frac{[-1]-[17]}{0-3} = \frac{-18}{-3} = 6$$

Note: Many students are concerned about determining which value of *x* should be used first. If you rework the problem, switching the values for *x* you will see that you get the same value in the end!

Example 2:

For a particular small town in west Texas, the population was noted over a five year period of time. The values are recorded on the following graph.



What was the average rate of change of population between 1980 and 1983?

Example 2 (Continued):

Solution:

$$\text{Average rate of change} = \frac{\text{population in 1983} - \text{population in 1980}}{1983 - 1980}$$

$$= \frac{f(1983) - f(1980)}{1983 - 1980}$$

$$= \frac{470 - 546}{1983 - 1980} = \frac{-76}{3} \approx -25.3$$

So the average rate of change of population between 1980 and 1983 was –25.3 people. Notice that the overall rate is negative, because the total population decreased over this period of time.