Review Exercise Set 3

Exercise 1: Determine the average rate of change of the given function over the interval from $x_1$ to $x_2$.

$$f(x) = 5x - 1; \ x_1 = 1, \ x_2 = 5$$

Exercise 2: Determine the average rate of change of the given function over the interval from $x_1$ to $x_2$.

$$f(x) = \frac{1}{2} x^2 - 3x - 1; \ x_1 = 1, \ x_2 = 4$$

Exercise 3: Find the average velocity for the given distance function and time interval.

$$s(t) = 16t^2; \ t = 3 \text{ to } t = 3.5$$
Exercise 4: Find the average velocity for the given distance function and time interval.

\[ s(t) = 1.5t^2 + 4t; \ t = 0 \text{ to } t = 2 \]

Exercise 5: The graph below shows the number of student computer visits in the SLAC lab between 1997 and 2007. Find the average rate of change for this time period.
Review Exercise Set 3 Answer Key

Exercise 1: Determine the average rate of change of the given function over the interval from $x_1$ to $x_2$.

$$f(x) = 5x - 1; \ x_1 = 1, \ x_2 = 5$$

Find $f(x_1)$ and $f(x_2)$

$$f(x_1) = 5(1) - 1$$
$$f(x_1) = 4$$

$$f(x_2) = 5(5) - 1$$
$$f(x_2) = 24$$

Find average rate of change

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{24 - 4}{5 - 1}$$
$$= \frac{20}{4}$$
$$= 5$$

Exercise 2: Determine the average rate of change of the given function over the interval from $x_1$ to $x_2$.

$$f(x) = \frac{1}{2}x^2 - 3x - 1; \ x_1 = 1, \ x_2 = 4$$

Find $f(x_1)$ and $f(x_2)$

$$f(x_1) = \frac{1}{2}(1)^2 - 3(1) - 1$$
$$f(x_1) = \frac{1}{2} - 3 - 1$$
$$f(x_1) = -3 \frac{1}{2} = -\frac{7}{2}$$

$$f(x_2) = \frac{1}{2}(4)^2 - 3(4) - 1$$
$$f(x_2) = 8 - 12 - 1$$
$$f(x_2) = -5$$
Exercise 2 (Continued):

Find average rate of change

\[ \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{-5 - \left(-\frac{7}{2}\right)}{4 - 1} \]
\[ = \frac{-5 + \frac{7}{2}}{3} \]
\[ = \frac{-\frac{3}{2}}{3} \]
\[ = -\frac{1}{2} \]

Exercise 3: Find the average velocity for the given distance function and time interval.

\[ s(t) = 16t^2; \quad t = 3 \text{ to } t = 3.5 \]

Find \( s(3) \) and \( s(3.5) \)

\[ s(3) = 16(3)^2 \]
\[ s(3) = 16(9) \]
\[ s(3) = 144 \]

\[ s(3.5) = 16(3.5)^2 \]
\[ s(3.5) = 16(12.25) \]
\[ s(3.5) = 196 \]

Find average velocity

\[ \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(3.5) - s(3)}{3.5 - 3} \]
\[ = \frac{196 - 144}{0.5} \]
\[ = \frac{52}{0.5} \]
\[ = 104 \]
Exercise 4: Find the average velocity for the given distance function and time interval.

\[ s(t) = 1.5t^2 + 4t; \quad t = 0 \text{ to } t = 2 \]

Find \( s(0) \) and \( s(2) \)

\[
\begin{align*}
  s(0) &= 1.5(0)^2 + 4(0) \\
  s(0) &= 0 \\
  s(2) &= 1.5(2)^2 + 4(2) \\
  s(2) &= 6 + 8 \\
  s(2) &= 14
\end{align*}
\]

Find average velocity

\[
\frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(2) - s(0)}{2 - 0}
\]

\[
\begin{align*}
  &= \frac{14 - 0}{2} \\
  &= \frac{14}{2} \\
  &= 7
\end{align*}
\]
Exercise 5: The graph below shows the number of student computer visits in the SLAC lab between 1997 and 2007. Find the average rate of change for this time period.

Identify the two points on the graph

(1997, 54,734) and (2007, 105,101)

Find average rate of change

\[
\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{105,101 - 54,734}{2007 - 1997} = \frac{50,367}{10} = 5,036.7
\]