Review Exercise Set 6

Exercise 1: Determine if the given functions are inverses of each other.

\[ f(x) = 3x + 1 \] and \[ g(x) = \frac{x - 1}{3} \]

Exercise 2: Find an equation for the inverse of the given function.

\[ f(x) = \frac{x}{3x - 6} \]

Exercise 3: Determine if the function in the graph below has an inverse function.
Exercise 4: Use the graph of \( f(x) \) to graph its inverse.

Exercise 5: Find the inverse of the given function with the stated domain restriction.

\[ f(x) = 3(x + 1)^2 - 1; \text{ where } x \geq -1 \]
Review Exercise Set 6 Answer Key

Exercise 1: Determine if the given functions are inverses of each other.

\[ f(x) = 3x + 1 \text{ and } g(x) = \frac{x - 1}{3} \]

Check if \( f\left[ g(x) \right] = x \)

\[
\begin{align*}
 f\left[ g(x) \right] &= x \\
 f\left[ \frac{x - 1}{3} \right] &= x \\
 3\left( \frac{x - 1}{3} \right) + 1 &= x \\
 x - 1 + 1 &= x \\
 x &= x
\end{align*}
\]

Check if \( g\left[ f(x) \right] = x \)

\[
\begin{align*}
 g\left[ f(x) \right] &= x \\
 g\left[ 3x + 1 \right] &= x \\
 \frac{(3x + 1) - 1}{3} &= x \\
 \frac{3x}{3} &= x \\
 x &= x
\end{align*}
\]

\( f(x) \) and \( g(x) \) are inverse functions

Exercise 2: Find an equation for the inverse of the given function.

\[ f(x) = \frac{x}{3x - 6} \]

Replace \( f(x) \) with \( y \)

\[ y = \frac{x}{3x - 6} \]
Exercise 2 (Continued):

Switch x and y

\[
x = \frac{y}{3y - 6}
\]

Solve for y

\[
x(3y - 6) = \frac{y}{3y - 6} (3y - 6)
\]

\[
3xy - 6x = y
\]

\[
3xy - y = 6x
\]

\[
y(3x - 1) = 6x
\]

\[
y = \frac{6x}{3x - 1}
\]

Replace y with \( f^{-1}(x) \)

\[
f^{-1}(x) = \frac{6x}{3x - 1}
\]

Exercise 3: Determine if the function in the graph below has an inverse function.

The function does not have an inverse because it fails the horizontal line test.
Exercise 4: Use the graph of $f(x)$ to graph its inverse.

- Graph the diagonal line $y = x$
- Reflect the function across the diagonal line
Exercise 5: Find the inverse of the given function with the stated domain restriction.

\[ f(x) = 3(x + 1)^2 - 1; \text{ where } x \geq -1 \]

Replace \( f(x) \) with \( y \)

\[ y = 3(x + 1)^2 - 1; \text{ where } x \geq -1 \]

Switch \( x \) and \( y \)

\[ x = 3(y + 1)^2 - 1; \text{ where } y \geq -1 \]

Solve for \( y \)

\[
\frac{x + 1}{3} = (y + 1)^2
\]

Since \( y \) must be greater than or equal to -1 take only the principal square root

\[
\sqrt{\frac{x + 1}{3}} = y + 1
\]

\[
\sqrt{\frac{x + 1}{3}} - 1 = y
\]

Replace \( y \) with \( f^{-1}(x) \)

\[
f^{-1}(x) = \sqrt{\frac{x + 1}{3}} - 1
\]