

## Review Exercise Set 20

Exercise 1: Use Gaussian elimination to find the solution for the given system of equations.

$$3x + y - z = 1$$

$$x - y + z = -3$$

$$2x + y + z = 0$$

Exercise 2: Use Gaussian elimination to find the solution for the given system of equations.

$$2x + 5y = 9$$

$$x + 2y - z = 3$$

$$-3x - 4y + 7z = 1$$

Exercise 3: The circle given by the equation  $x^2 + y^2 + ax + by + c = 0$  passes through the points  $(-2, 0)$ ,  $(-1, 7)$ , and  $(5, -1)$ . Find  $a$ ,  $b$ , and  $c$ .

## Review Exercise Set 20 Answer Key

Exercise 1: Use Gaussian elimination to find the solution for the given system of equations.

$$\begin{aligned}3x + y - z &= 1 \\x - y + z &= -3 \\2x + y + z &= 0\end{aligned}$$

Setup the augmented matrix

$$\left[ \begin{array}{ccc|c} 3 & 1 & -1 & 1 \\ 1 & -1 & 1 & -3 \\ 2 & 1 & 1 & 0 \end{array} \right]$$

Perform row operations to reduce the matrix

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 3 & 1 & -1 & 1 \\ 2 & 1 & 1 & 0 \end{array} \right] R_1 \leftrightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 4 & -4 & 10 \\ 2 & 1 & 1 & 0 \end{array} \right] -3R_1 + R_2 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 4 & -4 & 10 \\ 0 & 3 & -1 & 6 \end{array} \right] -2R_1 + R_3 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 3 & -1 & 6 \end{array} \right] R_2 \div 4 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 3 & -1 & 6 \end{array} \right] R_2 + R_1 \rightarrow R_1$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{2} \\ 0 & 0 & 2 & -\frac{3}{2} \end{array} \right] -3R_2 + R_3 \rightarrow R_3$$

Exercise 1 (Continued):

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -1 & \frac{5}{4} \\ 0 & 0 & 1 & -\frac{3}{4} \end{array} \right] R_3 \div 2 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{7}{4} \\ 0 & 0 & 1 & -\frac{3}{4} \end{array} \right] R_3 + R_2 \rightarrow R_2$$

The solution set  $(x, y, z)$  is  $\left(-\frac{1}{2}, \frac{7}{4}, -\frac{3}{4}\right)$

Exercise 2: Use Gaussian elimination to find the solution for the given system of equations.

$$\begin{aligned} 2x + 5y &= 9 \\ x + 2y - z &= 3 \\ -3x - 4y + 7z &= 1 \end{aligned}$$

Setup the augmented matrix

$$\left[ \begin{array}{ccc|c} 2 & 5 & 0 & 9 \\ 1 & 2 & -1 & 3 \\ -3 & -4 & 7 & 1 \end{array} \right]$$

Perform row operations to reduce the matrix

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & 5 & 0 & 9 \\ -3 & -4 & 7 & 1 \end{array} \right] R_1 \leftrightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 3 \\ -3 & -4 & 7 & 1 \end{array} \right] -2R_1 + R_2 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 10 \end{array} \right] 3R_1 + R_3 \rightarrow R_3$$

Exercise 2 (Continued):

$$\left[ \begin{array}{ccc|c} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 10 \end{array} \right] -2R_2 + R_1 \rightarrow R_1$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -5 & -3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{array} \right] -2R_2 + R_3 \rightarrow R_3$$

$0 \neq 4$  so this system is inconsistent and has no solution.

Exercise 3: The circle given by the equation  $x^2 + y^2 + ax + by + c = 0$  passes through the points  $(-2, 0)$ ,  $(-1, 7)$ , and  $(5, -1)$ . Find  $a$ ,  $b$ , and  $c$ .

Substitute the given points into the equation to find the system of equations

$$\begin{aligned} x^2 + y^2 + ax + by + c &= 0 \\ (-2)^2 + (0)^2 + a(-2) + b(0) + c &= 0 \\ 4 - 2a + c &= 0 \\ -2a + c &= -4 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 + ax + by + c &= 0 \\ (-1)^2 + (7)^2 + a(-1) + b(7) + c &= 0 \\ 1 + 49 - a + 7b + c &= 0 \\ -a + 7b + c &= -50 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 + ax + by + c &= 0 \\ (5)^2 + (-1)^2 + a(5) + b(-1) + c &= 0 \\ 25 + 1 + 5a - b + c &= 0 \\ 5a - b + c &= -26 \end{aligned}$$

$$\begin{aligned} -2a + c &= -4 \\ -a + 7b + c &= -50 \\ 5a - b + c &= -26 \end{aligned}$$

Setup the augmented matrix

$$\left[ \begin{array}{ccc|c} -2 & 0 & 1 & -4 \\ -1 & 7 & 1 & -50 \\ 5 & -1 & 1 & -26 \end{array} \right]$$

Exercise 3 (Continued):

Perform row operations to reduce the matrix

$$\left[ \begin{array}{ccc|c} -1 & 7 & 1 & -50 \\ -2 & 0 & 1 & -4 \\ 5 & -1 & 1 & -26 \end{array} \right] R_1 \leftrightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -7 & -1 & 50 \\ -2 & 0 & 1 & -4 \\ 5 & -1 & 1 & -26 \end{array} \right] -1R_1 \rightarrow R_1$$

$$\left[ \begin{array}{ccc|c} 1 & -7 & -1 & 50 \\ 0 & -14 & -1 & 96 \\ 5 & -1 & 1 & -26 \end{array} \right] 2R_1 + R_2 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -7 & -1 & 50 \\ 0 & -14 & -1 & 96 \\ 0 & 34 & 6 & -276 \end{array} \right] -5R_1 + R_3 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 1 & -7 & -1 & 50 \\ 0 & 20 & 5 & -180 \\ 0 & 34 & 6 & -276 \end{array} \right] R_3 + R_2 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -7 & -1 & 50 \\ 0 & 1 & \frac{1}{4} & -9 \\ 0 & 17 & 3 & -138 \end{array} \right] R_2 \div 20 \rightarrow R_2$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & \frac{3}{4} & -13 \\ 0 & 1 & \frac{1}{4} & -9 \\ 0 & 17 & 3 & -138 \end{array} \right] 7R_2 + R_1 \rightarrow R_1$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & \frac{3}{4} & -13 \\ 0 & 1 & \frac{1}{4} & -9 \\ 0 & 0 & -\frac{5}{4} & 15 \end{array} \right] -17R_2 + R_3 \rightarrow R_3$$

Exercise 3 (Continued):

$$\left[ \begin{array}{ccc|c} 1 & 0 & \frac{3}{4} & -13 \\ 0 & 1 & \frac{1}{4} & -9 \\ 0 & 0 & 1 & -12 \end{array} \right] -\frac{4}{3}R_3 \rightarrow R_3$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & \frac{1}{4} & -9 \\ 0 & 0 & 1 & -12 \end{array} \right] -\frac{3}{4}R_3 + R_1 \rightarrow R_1$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -12 \end{array} \right] -\frac{1}{4}R_3 + R_2 \rightarrow R_2$$

The solution set (a, b, c) is (-4, -6, -12)