Review Exercise Set 22

Exercise 1: Determine if matrix B is the multiplicative inverse of matrix A.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$$

Exercise 2: Find the inverse of A using the equation $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

Exercise 3: Find the inverse of A by using an augmented matrix and row operations.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ -1 & -1 & -2 \end{bmatrix}$$

Exercise 4: Write the given linear system as a matrix equation in the form AX = B, where A is the coefficient matrix and B is the constant matrix.

$$x + 2y - z = -4$$
$$x + 4y - 2z = -6$$
$$2x + 3y + z = 3$$

Exercise 5: Write the given linear system as a matrix equation in the form AX = B and solve the system.

$$5x - 4y = 2$$
$$6x - 5y = 1$$

Review Exercise Set 22 Answer Key

Exercise 1: Determine if matrix B is the multiplicative inverse of matrix A.

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$$

Determine if $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} (1)(3)+(-1)(2) & (1)(-1)+(-1)(-1) \\ (2)(3)+(-3)(2) & (2)(-1)+(-3)(-1) \end{bmatrix} \\ &= \begin{bmatrix} (3-2) & (-1+1) \\ (6-6) & (-2+3) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Determine if $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{aligned} BA &= \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} (3)(1)+(-1)(2) & (3)(-1)+(-1)(-3) \\ (2)(1)+(-1)(2) & (2)(-1)+(-1)(-3) \end{bmatrix} \\ &= \begin{bmatrix} (3-2) & (-3+3) \\ (2-2) & (-2+3) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

A and B are multiplicative inverses.

Exercise 2: Find the inverse of A using the equation $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

$$A = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

a = -5, b = 3, c = 2, d = -1

$$A^{-1} = \frac{1}{(-5)(-1)-(3)(2)}\begin{bmatrix} -1 & -3 \\ -2 & -5 \end{bmatrix}$$
$$= \frac{1}{5-6}\begin{bmatrix} -1 & -3 \\ -2 & -5 \end{bmatrix}$$
$$= -1\begin{bmatrix} -1 & -3 \\ -2 & -5 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

Exercise 3: Find the inverse of A by using an augmented matrix and row operations.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ -1 & -1 & -2 \end{bmatrix}$$

Setup the augmented matrix

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ -1 & -1 & -2 & 0 & 0 & 1 \end{array}\right]$$

Use row operations to reduce the left-hand side of the augmented matrix

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & 1 \end{array}\right] R_1 + R_3 \to R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 1 & 0 & 1 \end{array}\right] -2R_2 + R_1 \to R_1$$

Exercise 3 (Continued):

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -4 & 1 & -1 & 1 \end{array}\right] -1R_2 + R_3 \rightarrow R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array}\right] R_3 \div -4 \rightarrow R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & -\frac{5}{4} & -\frac{3}{4} \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array}\right] \begin{array}{l} 3R_3 + R_1 \rightarrow R_1 \\ \\ \\ \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{4} & -\frac{5}{4} & -\frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{array}\right] -1R_3 + R_2 \rightarrow R_2$$

The inverse matrix A^{-1} is $\begin{bmatrix} \frac{1}{4} & -\frac{5}{4} & -\frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{bmatrix}$

Exercise 4: Write the given linear system as a matrix equation in the form AX = B, where A is the coefficient matrix and B is the constant matrix.

x + 2y - z = -4
x + 4y - 2z = -6
2x + 3y + z = 3

$$AX = B$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 4 & -2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -6 \\ 3 \end{bmatrix}$$

Exercise 5: Write the given linear system as a matrix equation in the form AX = B and solve the system.

5x - 4y = 2
6x - 5y = 1

Write the system as a matrix equation

$$AX = B$$

$$\begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Solve the matrix equation for X

AX = B
$A^{-1}AX = A^{-1}B$
$X = A^{-1}B$

Find A^{-1}

$$\begin{aligned} A^{-1} &= \frac{1}{(5)(-5)-(-4)(6)} \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix} \\ &= \frac{1}{-25+24} \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix} \\ &= -1 \begin{bmatrix} -5 & 4 \\ -6 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix} \end{aligned}$$

Find X

$$X = A^{-1}B$$

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} (5)(2)+(-4)(1) \\ (6)(2)+(-5)(1) \end{bmatrix} \\ &= \begin{bmatrix} (10-4) \\ (12-5) \end{bmatrix} \\ &= \begin{bmatrix} 6 \\ 7 \end{bmatrix} \end{aligned}$$

The solution set (x, y) is (6, 7).