# Review Exercise Set 23

Exercise 1: Evaluate the following second-order determinant.

$$\begin{vmatrix} 2 & -4 \\ 7 & 3 \end{vmatrix}$$

Exercise 2: Use Cramer's Rule to solve the given system of equations.

2x - y = 5
-x + 2y = 3

Exercise 3: Evaluate the following third-order determinant.

$$\begin{vmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 7 & 0 & -1 \end{vmatrix}$$

Exercise 4: Use Cramer's Rule to solve the given system of equations.

$$-3x + y - z = 2$$
$$x + 2y - 3z = -6$$
$$2x - y + z = -1$$

Exercise 5: Evaluate the following fourth-order determinant.

$$\begin{vmatrix} 1 & 0 & 2 & -1 \\ 3 & 2 & 0 & 1 \\ 2 & 0 & 3 & -4 \\ 0 & 2 & -1 & 1 \end{vmatrix}$$

# Review Exercise Set 23 Answer Key

Exercise 1: Evaluate the following second-order determinant.

$$\begin{vmatrix} 2 & -4 \\ 7 & 3 \end{vmatrix}$$

= (2)(3) - (7)(-4)
= 6 + 28
= 34

Exercise 2: Use Cramer's Rule to solve the given system of equations.

2x - y = 5
-x + 2y = 3

Find the value of the determinants D, $D_x$, and $D_y$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} \quad D_x = \begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix} \quad D_y = \begin{vmatrix} 2 & 5 \\ -1 & 3 \end{vmatrix}$$

$$= (2)(2) - (-1)(-1) \quad = (5)(2) - (3)(-1) \quad = (2)(3) - (-1)(5)$$

$$= 4 - 1 \quad = 10 + 3 \quad = 6 + 5$$

$$= 3 \quad = 13 \quad = 11$$

Find the value of x and y

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$$

$$= \frac{13}{3} \quad = \frac{11}{3}$$

The solution (x, y) to the system of equation is $\left(\frac{13}{3}, \frac{11}{3}\right)$.

Exercise 3: Evaluate the following third-order determinant.

$$\begin{vmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 7 & 0 & -1 \end{vmatrix}$$

Choose a row or column to use to expand the third-order determinant into a sum of second-order determinants (or minors)

Since column one contains a 0 as one of its elements, we will use this column.

Identify the signs to be multiplied with the elements used from the first column to expand the determinant.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Since we used the first column the signs are: +, -, and +.

Identify the minors for the elements of column one

For each element in the first column, you can find its minor by eliminating the column and row that it is in. The remaining element will be the minor.

For the element in row 1 column 1 $\begin{vmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 7 & 0 & -1 \end{vmatrix}$ the minor is $\begin{vmatrix} 1 & -3 \\ 0 & -1 \end{vmatrix}$

For the element in row 2 column 1 $\begin{vmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 7 & 0 & -1 \end{vmatrix}$ the minor is $\begin{vmatrix} -2 & 4 \\ 0 & -1 \end{vmatrix}$

For the element in row 3 column 1 $\begin{vmatrix} 3 & -2 & 4 \\ 0 & 1 & -3 \\ 7 & 0 & -1 \end{vmatrix}$ the minor is $\begin{vmatrix} -2 & 4 \\ 1 & -3 \end{vmatrix}$

Exercise 3 (Continued):

Expand the third-order determinant into its minors

$$=(1)(3)\begin{vmatrix}1 & -3\\0 & -1\end{vmatrix}+(-1)(0)\begin{vmatrix}-2 & 4\\0 & -1\end{vmatrix}+(1)(7)\begin{vmatrix}-2 & 4\\1 & -3\end{vmatrix}$$

Evaluate

= 3[(1)(-1) - (0)(-3)] - 0 + 7[(-2)(-3) - (1)(4)]
= 3[-1 + 3] + 7[6 - 4]
= 3[2] + 7[2]
= 6 + 14
= 20

Exercise 4: Use Cramer's Rule to solve the given system of equations.

-3x + y - z = 2
x + 2y - 3z = -6
2x - y + z = -1

Find the value of the determinants D, $D_x$ , $D_y$ , and $D_z$

$$D = \begin{vmatrix}-3 & 1 & -1\\1 & 2 & -3\\2 & -1 & 1\end{vmatrix}$$

$$D = (1)(-3)\begin{vmatrix}2 & -3\\-1 & 1\end{vmatrix} + (-1)(1)\begin{vmatrix}1 & -1\\-1 & 1\end{vmatrix} + (1)(2)\begin{vmatrix}1 & -1\\2 & -3\end{vmatrix}$$

D = -3[(2)(1) - (-1)(-3)] - 1[(1)(1) - (-1)(-1)] + 2[(1)(-3) - (2)(-1)]
D = -3[2 - 3] - 1[1 - 1] + 2[-3 + 2]
D = -3[-1] - 1[0] + 2[-1]
D = 3 - 0 - 2
D = 1

$$D_x = \begin{vmatrix}2 & 1 & -1\\-6 & 2 & -3\\-1 & -1 & 1\end{vmatrix}$$

$$D_x = (1)(2)\begin{vmatrix}2 & -3\\-1 & 1\end{vmatrix} + (-1)(-6)\begin{vmatrix}1 & -1\\-1 & 1\end{vmatrix} + (1)(-1)\begin{vmatrix}1 & -1\\2 & -3\end{vmatrix}$$

$D_x$ = 2[-1] + 6[0] - 1[-1]
$D_x$ = -2 + 0 + 1
$D_x$ = -1

Exercise 4 (Continued):

$$D_y = \begin{vmatrix} -3 & 2 & -1 \\ 1 & -6 & -3 \\ 2 & -1 & 1 \end{vmatrix}$$

$$D_y = (1)(-3)\begin{vmatrix} -6 & -3 \\ -1 & 1 \end{vmatrix} + (-1)(1)\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} + (1)(2)\begin{vmatrix} 2 & -1 \\ -6 & -3 \end{vmatrix}$$

$D_y$ = -3[(-6)(1) - (-1)(-3)] - 1[(2)(1) - (-1)(-1)] + 2[(2)(-3) - (-6)(-1)]
$D_y$ = -3[-6 - 3] - 1[2 - 1] + 2[-6 - 6]
$D_y$ = -3[-9] - 1[1] + 2[-12]
$D_y$ = 27 - 1 - 24
$D_y$ = 2

$$D_z = \begin{vmatrix} -3 & 1 & 2 \\ 1 & 2 & -6 \\ 2 & -1 & -1 \end{vmatrix}$$

$$D_z = (1)(-3)\begin{vmatrix} 2 & -6 \\ -1 & -1 \end{vmatrix} + (-1)(1)\begin{vmatrix} 1 & 2 \\ -1 & -1 \end{vmatrix} + (1)(2)\begin{vmatrix} 1 & 2 \\ 2 & -6 \end{vmatrix}$$

$D_z$ = -3[(2)(-1) - (-1)(-6)] - 1[(1)(-1) - (-1)(2)] + 2[(1)(-6) - (2)(2)]
$D_z$ = -3[-2 - 6] - 1[-1 + 2] + 2[-6 - 4]
$D_z$ = -3[-8] - 1[1] + 2[-10]
$D_z$ = 24 - 1 - 20
$D_z$ = 3

Find the value of x, y, and z

$$x = \frac{D_x}{D} = \frac{-1}{1} = -1 \qquad y = \frac{D_y}{D} = \frac{2}{1} = 2 \qquad z = \frac{D_z}{D} = \frac{3}{1} = 3$$

The solution (x, y, z) to the system of equation is (-1, 2, 3).

Exercise 5: Evaluate the following fourth-order determinant.

$$\begin{vmatrix} 1 & 0 & 2 & -1 \\ 3 & 2 & 0 & 1 \\ 2 & 0 & 3 & -4 \\ 0 & 2 & -1 & 1 \end{vmatrix}$$

Expand the fourth-order determinant using the row or column that has the most 0's as elements

$$= (-1)(0)\begin{vmatrix} 3 & 0 & 1 \\ 2 & 3 & -4 \\ 0 & -1 & 1 \end{vmatrix} + (1)(2)\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -4 \\ 0 & -1 & 1 \end{vmatrix} + (-1)(0)\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} + (1)(2)\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= 0 + 2\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -4 \\ 0 & -1 & 1 \end{vmatrix} + 0 + 2\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -4 \\ 0 & -1 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 2 & 3 & -4 \end{vmatrix}$$

Expand the third-order determinants using the row or column that has the most 0's as elements

$$= 2[(1)(1)\begin{vmatrix} 3 & -4 \\ -1 & 1 \end{vmatrix} + (-1)(2)\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} + (1)(0)\begin{vmatrix} 2 & -1 \\ 3 & -4 \end{vmatrix}] +$$

$$2[(-1)(2)\begin{vmatrix} 3 & 1 \\ 2 & -4 \end{vmatrix} + (1)(0)\begin{vmatrix} 1 & -1 \\ 2 & -4 \end{vmatrix} + (-1)(3)\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}]$$

$$= 2[\begin{vmatrix} 3 & -4 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} + 0] + 2[-2\begin{vmatrix} 3 & 1 \\ 2 & -4 \end{vmatrix} + 0 - 3\begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix}]$$

= 2[(3)(1) - (-1)(-4) - 2[(2)(1) - (-1)(-1)] ] +
2[-2[(3)(-4) - (2)(1)] - 3[(1)(1) - (3)(-1)] ]

= 2[3 - 4 - 2(2 - 1)] + 2[-2(-12 - 2) - 3(1 + 3)]
= 2[-1 - 2(1)] + 2[-2(-14) - 3(4)]
= 2[-1 - 2] + 2[28 - 12]
= 2[-3] + 2[16]
= -6 + 32
= 26