Review Exercise Set 11

Exercise 1: List all of the possible rational zeros for the given polynomial.

$p(x) = 3x^5 + 2x^4 - 5x^3 + x - 10$

Exercise 2: List all of the possible rational zeros for the given polynomial. Then use synthetic division to locate one of the zeros. Use the quotient to find the remaining zeros.

$p(x) = x^3 - 6x^2 + 11x - 6$

Exercise 3: Find the polynomial function with real coefficients that satisfies the given conditions.

degree = 4; zeros include -1, $\frac{3}{2}$, and 1 + i; p(1) = -2

Exercise 4: Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for the given polynomial.

$$p(x) = x^4 - 5x^3 + 5x^2 + 25x - 26$$

Exercise 5: Find all zeros of the given polynomial by using the Rational Zero Theorem, Descartes's Rule of Signs, and synthetic division.

$$p(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$$

Review Exercise Set 11 Answer Key

Exercise 1: List all of the possible rational zeros for the given polynomial.

$p(x) = 3x^5 + 2x^4 - 5x^3 + x - 10$

List the factors of the constant, -10

p = ±1, ±2, ±5, ±10

List the factors of the leading coefficient, 3

q = ±1, ±3

Divide p by q

$$\frac{p}{q} = \frac{\pm1, \pm2, \pm5, \pm10}{\pm1, \pm3}$$
$$= \frac{\pm1}{\pm1}, \frac{\pm2}{\pm1}, \frac{\pm5}{\pm1}, \frac{\pm10}{\pm1}, \frac{\pm1}{\pm3}, \frac{\pm2}{\pm3}, \frac{\pm5}{\pm3}, \frac{\pm10}{\pm3}$$
$$= \pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$$

List from smallest to largest

$$\frac{p}{q} = \pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm\frac{5}{3}, \pm2, \pm\frac{10}{3}, \pm5, \pm10$$

Exercise 2: List all of the possible rational zeros for the given polynomial. Then use synthetic division to locate one of the zeros. Use the quotient to find the remaining zeros.

$p(x) = x^3 - 6x^2 + 11x - 6$

Possible rational zeros

p = ±1, ±2, ±3, ±6
q = ±1

$$\frac{p}{q} = \frac{\pm1, \pm2, \pm3, \pm6}{\pm1}$$
$$= \frac{\pm1}{\pm1}, \frac{\pm2}{\pm1}, \frac{\pm3}{\pm1}, \frac{\pm6}{\pm1}$$
$$= \pm1, \pm2, \pm3, \pm6$$

Exercise 2 (Continued):

Use synthetic division to locate a zero

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

Since the remainder is zero, x = 1 is a zero of the polynomial. The resulting coefficients of 1, -5, and 6 will be used for our quotient polynomial.

Write the polynomial in factored form

First we will find our factor for the zero of 1

x = 1
x - 1 = 0

Now, use the coefficients of 1, -5, and 6 for the quotient polynomial which will have a degree that is one less than the non-factored polynomial.

$p(x) = (x - 1)(x^2 - 5x + 6)$

Factor the quadratic term

$p(x) = (x - 1)(x - 2)(x - 3)$

Exercise 3: Find the polynomial function with real coefficients that satisfies the given conditions.

degree = 4; zeros include -1, $\frac{3}{2}$, and 1 + i; p(1) = -2

Find the factors for the given zeros

x = -1
x + 1 = 0

$x = \frac{3}{2}$
2x = 3
2x - 3 = 0

x = 1 + i
x - 1 - i = 0

x = 1 - i
x - 1 + i = 0

Remember, imaginary roots must come in conjugate pairs. So, if 1 + i is a zero then 1 - i must also be a zero.

Exercise 3 (Continued):

Use the linear factorization theorem to setup the polynomial function

$p(x) = a_n(x - c_1)(x - c_2)(x - c_3)(x - c_4)$
$p(x) = a_n(x + 1)(2x - 3)(x - 1 - i)(x - 1 + i)$
$p(x) = a_n(x + 1)(2x - 3)[(x - 1) - i][(x - 1) + i]$
$p(x) = a_n(x + 1)(2x - 3)[(x - 1)^2 - i^2]$
$p(x) = a_n(2x^2 - x - 3)(x^2 - 2x + 1 - (-1))$
$p(x) = a_n(2x^2 - x - 3)(x^2 - 2x + 1 + 1)$
$p(x) = a_n(2x^2 - x - 3)(x^2 - 2x + 2)$
$p(x) = a_n(2x^4 - 5x^3 + 3x^2 + 4x - 6)$

Use $p(1) = -2$ to find the value of a_n

$p(1) = a_n(2(1)^4 - 5(1)^3 + 3(1)^2 + 4(1) - 6)$
$-2 = a_n(2 - 5 + 3 + 4 - 6)$
$-2 = -2a_n$
$1 = a_n$

Substitute the value of a into the polynomial function

$p(x) = (1)(2x^4 - 5x^3 + 3x^2 + 4x - 6)$
$p(x) = 2x^4 - 5x^3 + 3x^2 + 4x - 6$

Exercise 4: Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for the given polynomial.

$p(x) = x^4 - 5x^3 + 5x^2 + 25x - 26$

Positive real zeros

Count the sign changes in the polynomial

x^4 to $-5x^3$ = 1st sign change
$-5x^3$ to $5x^2$ = 2nd sign change
$25x$ to -26 = 3rd sign change

Since there are 3 sign changes the possible number of positive real zeros are either 3 or 1.

Exercise 4 (Continued):

Negative real zeros

Find p(-x) and then count the sign changes

$p(-x) = (-x)^4 - 5(-x)^3 + 5(-x)^2 + 25(-x) - 26$
$p(-x) = x^4 + 5x^3 + 5x^2 - 25x - 26$

$5x^2$ to $-25x$ = 1st sign change

Since there is only 1 sign change the possible number of negative real zeros is 1.

Exercise 5: Find all zeros of the given polynomial by using the Rational Zero Theorem, Descartes's Rule of Signs, and synthetic division.

$p(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$

Possible rational zeros

$p = \pm1, \pm2, \pm4$
$q = \pm1$

$$\frac{p}{q} = \frac{\pm1, \pm2, \pm4}{\pm1}$$
$$= \frac{\pm1}{\pm1}, \frac{\pm2}{\pm1}, \frac{\pm4}{\pm1}$$
$$= \pm1, \pm2, \pm4$$

Positive real zeros

Since there are 4 sign changes the possible number of positive real zeros are either 4, 2, or 0.

Negative real zeros

$p(-x) = (-x)^4 - 2(-x)^3 + 5(-x)^2 - 8(-x) + 4$
$p(-x) = x^4 + 2x^3 + 5x^2 + 8x + 4$

Since there are no sign changes there are no possible negative real zeros.

Exercise 5 (Continued):

Use synthetic division and only the positive possible rational zeros to locate a zero since there are no negative real zeros.

1	1	−2	5	−8	4
		1	−1	4	−4
	1	−1	4	−4	0

1 is a zero of the polynomial since the remainder is zero. We can now use these resulting coefficients to continue to find zeros of the polynomial.

$p(x) = (x - 1)(x^3 - x^2 + 4x - 4)$

1	1	−1	4	−4
		1	0	4
	1	0	4	0

1 is again a zero of the polynomial so it would have a multiplicity of 2. The resulting polynomial is now reduced to a quadratic equation so we can stop with the synthetic division and solve for the remaining zeros by either factoring or the quadratic formula.

$p(x) = (x - 1)^2(x^2 + 4)$

The factor $x^2 + 4$ cannot be factored so we would set it equal to zero and then solve for x to find the remaining zeros.

$x^2 + 4 = 0$
$x^2 = -4$
$x = \pm 2i$

The zeros of the polynomial are 1 (multiplicity of 2), -2i, and 2i.