Review Exercise Set 12

Exercise 1: Find the domain of the given rational function.

\[ h(x) = \frac{x - 5}{x^2 + 4x + 3} \]

Exercise 2: Use the given graph to complete the statements below.

a) \( \text{As } x \to -\infty, f(x) \to \)
b) \( \text{As } x \to -2^-, f(x) \to \)
c) \( \text{As } x \to -2^+, f(x) \to \)
d) \( \text{As } x \to 2^-, f(x) \to \)
e) \( \text{As } x \to 2^+, f(x) \to \)
f) \( \text{As } x \to \infty, f(x) \to \)
Exercise 3: Find the vertical and horizontal asymptotes of the given rational function.

\[ g(x) = \frac{3x + 7}{x^2 - x - 6} \]

Exercise 4: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

\[ r(x) = \frac{2x^2}{x^2 + 4x - 12} \]
Exercise 5: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

\[ r(x) = \frac{x^3 + 2}{x^2 + x} \]
Review Exercise Set 12 Answer Key

Exercise 1: Find the domain of the given rational function.

\[ h(x) = \frac{x - 5}{x^2 + 4x + 3} \]

Set the denominator equal to zero and solve for \( x \)

\[ x^2 + 4x + 3 = 0 \]
\[ (x + 1)(x + 3) = 0 \]
\[ x + 1 = 0 \text{ or } x + 3 = 0 \]
\[ x = -1 \text{ or } x = -3 \]

Exclude the values that make the denominator zero from the domain

Domain: \( (-\infty, -3) \cup (-3, -1) \cup (-1, \infty) \)

Exercise 2: Use the given graph to complete the statements below.

a) As \( x \to -\infty \), \( f(x) \to \)
b) As \( x \to -2^- \), \( f(x) \to \)
c) As \( x \to -2^+ \), \( f(x) \to \)
d) As \( x \to 2^- \), \( f(x) \to \)
e) As \( x \to 2^+ \), \( f(x) \to \)
f) As \( x \to \infty \), \( f(x) \to \)
Exercise 3: Find the vertical and horizontal asymptotes of the given rational function.

\[ g(x) = \frac{3x + 7}{x^2 - x - 6} \]

Vertical asymptote

Set the denominator equal to zero and solve for x

\[ x^2 - x - 6 = 0 \]

\[ (x + 2)(x - 3) = 0 \]

\[ x + 2 = 0 \text{ or } x - 3 = 0 \]
\[ x = -2 \text{ or } x = 3 \]

The vertical asymptotes will be at \( x = -2 \) and \( x = 3 \)

Horizontal asymptote

Compare degrees of the numerator and denominator

degree of numerator: 1
degree of denominator: 2

Since the denominator has a larger degree, the horizontal asymptote will be the x-axis or \( y = 0 \).

Exercise 4: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

\[ r(x) = \frac{2x^2}{x^2 + 4x - 12} \]

Vertical asymptote

\[ x^2 + 4x - 12 = 0 \]
\[ (x + 6)(x - 2) = 0 \]
\[ x + 6 = 0 \text{ or } x - 2 = 0 \]
\[ x = -6 \text{ or } x = 2 \]

Horizontal asymptote

degree of numerator: 2
degree of denominator: 2

degrees are the same so the horizontal asymptote will be the ratio of the leading coefficients.

\[ y = \frac{2}{1} = 2 \]
Exercise 4 (Continued):

Intercepts

Let \( x = 0 \)

\[
\begin{align*}
  r(0) &= \frac{2(0)^2}{(0)^2 + 4(0) - 12} \\
  r(0) &= 0
\end{align*}
\]

Let \( r(x) = 0 \)

\[
\begin{align*}
  0 &= \frac{2x^2}{x^2 + 4x - 12} \\
  0 &= 2x^2 \\
  0 &= x^2 \\
  0 &= x
\end{align*}
\]

y-intercept (0, 0) \hspace{1cm} \text{x-intercept (0, 0)}

Symmetry

\[
\begin{align*}
  r(-x) &= r(x) \\
  \frac{2(-x)^2}{(-x)^2 + 4(-x) - 12} &= \frac{2x^2}{x^2 + 4x - 12} \\
  \frac{2x^2}{x^2 - 4x - 12} &= \frac{2x^2}{x^2 + 4x - 12}
\end{align*}
\]

\( r(-x) \) and \( r(x) \) are not the same functions so it is not symmetric about the y-axis.

\[
\begin{align*}
  r(-x) &= -r(x) \\
  \frac{2(-x)^2}{(-x)^2 + 4(-x) - 12} &= -\frac{2x^2}{x^2 + 4x - 12} \\
  \frac{2x^2}{x^2 - 4x - 12} &= \frac{-2x^2}{x^2 + 4x - 12}
\end{align*}
\]

\( r(-x) \) and \( -r(x) \) are not the same functions so it is not symmetric about the origin.
Exercise 4 (Continued):

Graph

Exercise 5: Graph the given rational function by finding any symmetry, intercepts, asymptotes, and any additional points.

\[ r(x) = \frac{x^3 + 2}{x^2 + x} \]

Vertical asymptote

\[ x^2 + x = 0 \]
\[ x(x + 1) = 0 \]
\[ x = 0 \text{ or } x + 1 = 0 \]
\[ x = 0 \text{ or } x = -1 \]

Horizontal asymptote

degree of numerator: 3
degree of denominator: 2

The numerator has the larger degree so there is no horizontal asymptote. However, since the difference in the degrees is 1 there will be a slant asymptote.
Exercise 5 (Continued):

Slant asymptote

Divide the rational function using long division

\[
x^2 + x \overline{\frac{x^3}{x^3 + 0x^2 + 0x + 2}}
\]

\[
x^3 + x^2
\]

\[
- x^2 + 0x
\]

\[
- x^2 - x
\]

\[
x + 2
\]

The quotient of \( x - 1 \) is the slant asymptote.

Intercepts

Let \( r(x) = 0 \)

Let \( x = 0 \)

\( x \) cannot be zero since this is the location of one of the vertical asymptotes

\[
0 = \frac{x^3 + 2}{x^2 + x}
\]

\[
0 = x^3 + 2
\]

\[
-2 = x^3
\]

\[
-\sqrt{2} = x
\]

\[
-1.26 \approx x
\]

\( x \)-intercept \((-\sqrt{2}, 0)\)

Symmetry

\[
r(-x) = r(x)
\]

\[
\frac{(-x)^3 + 2}{(-x)^3 + (-x)} = \frac{x^3 + 2}{x^3 + 2}
\]

\[
\frac{-x^3 + 2}{x^2 - x} = \frac{x^3 + 2}{x^2 + x}
\]

\( r(-x) \) and \( r(x) \) are not the same functions so it is not symmetric about the y-axis.
Exercise 5 (Continued):

\[ r(-x) = -r(x) \]

\[
\frac{(-x)^3 + 2}{(-x)^2 + (-x)} = -\frac{x^3 + 2}{x^2 + x}
\]

\[
\frac{-x^3 + 2}{x^2 - x} = -\frac{x^3 - 2}{x^2 + x}
\]

r(-x) and -r(x) are not the same functions so it is not symmetric about the origin.

Graph