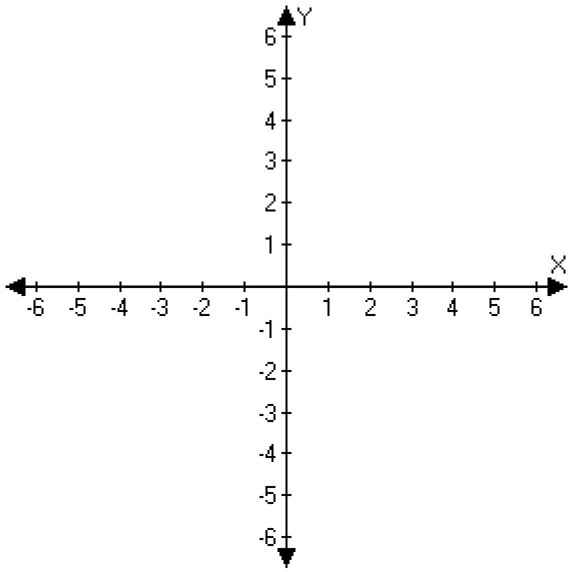
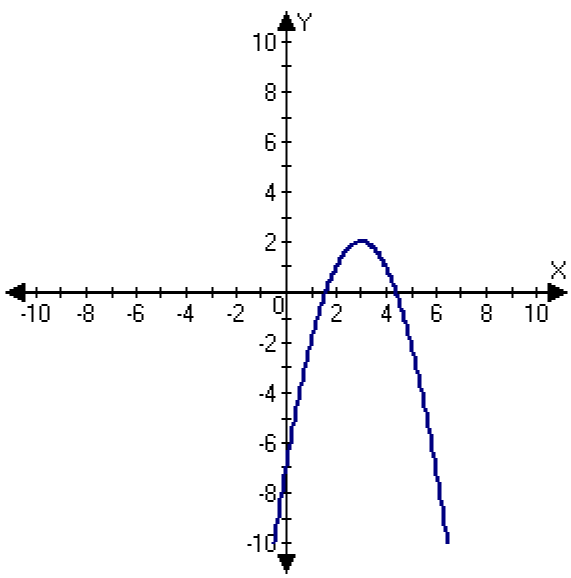
Review Exercise Set 8

Exercise 1: Graph the given quadratic function written in standard form.

$f(x) = (x + 2)^2 - 1$



Exercise 2: Use the given graph of a quadratic function to write the function's equation in standard form.



Exercise 3: Find the coordinates of the vertex for the parabola defined by the given quadratic function.

$g(x) = 2x^2 - 7x - 4$

Exercise 4: Sketch the graph of the given quadratic function by first finding the vertex, intercepts, and axis of symmetry.

$s(t) = -t^2 + 4t + 10$

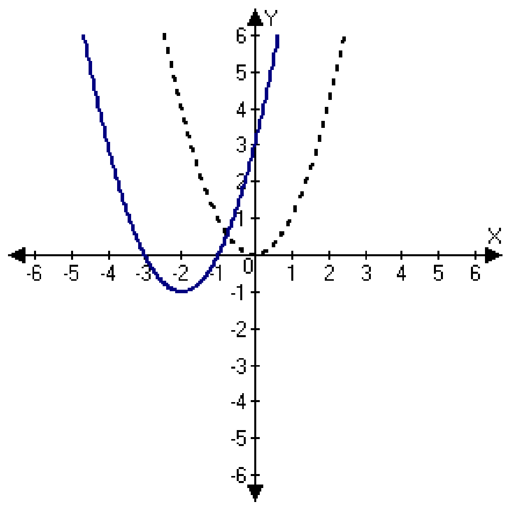
Exercise 5: A farmer has 1200 feet of fencing that he can use to enclose a rectangular lot and split it in two by another fence that runs parallel to one side of the lot. Find the dimensions of the lot that maximize the total enclosed area. What is the maximum area?

Review Exercise Set 8 Answer Key

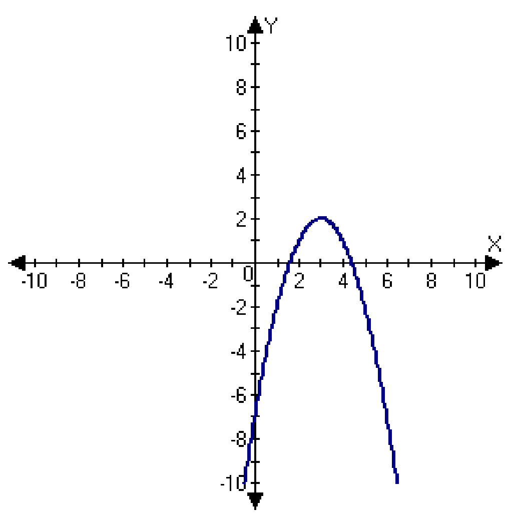
Exercise 1: Graph the given quadratic function written in standard form.

$f(x) = (x + 2)^2 - 1$

We can graph this function by starting off with the standard quadratic function of x and then use the transformations (horizontal shift left 2 units and vertical shift down 1 unit).



Exercise 2: Use the given graph of a quadratic function to write the function's equation in standard form.



The quadratic function in the graph contains the following transformations:

reflected downward
shifted horizontally to the right 3 units
shifted vertically up 2 units

The equation of the function would be $f(x) = -(x - 3)^2 + 2$

Exercise 3: Find the coordinates of the vertex for the parabola defined by the given quadratic function.

$g(x) = 2x^2 - 7x - 4$

Find the x-coordinate of the vertex

$$
\begin{aligned}
x &= \frac{-b}{2a} \\
&= -\frac{-7}{2(2)} \\
&= \frac{7}{4}
\end{aligned}
$$

Find the y-coordinate of the vertex

$$
\begin{aligned}
g\left(\frac{7}{4}\right) &= 2\left(\frac{7}{4}\right)^2 - 7\left(\frac{7}{4}\right) - 4 \\
&= 2\left(\frac{49}{16}\right) - \frac{49}{4} - 4 \\
&= \frac{49}{8} - \frac{98}{8} - \frac{32}{8} \\
&= -\frac{81}{8}
\end{aligned}
$$

The vertex is $\left(\frac{7}{4}, -\frac{81}{8}\right)$

Exercise 4: Sketch the graph of the given quadratic function by first finding the vertex, intercepts, and axis of symmetry.

$s(t) = -t^2 + 4t + 10$

Find the x-coordinate of the vertex

$$
\begin{aligned}
t &= \frac{-b}{2a} \\
&= -\frac{4}{2(-1)} \\
&= 2
\end{aligned}
$$

Exercise 4 (Continued):

Find the y-coordinate of the vertex

$s(2) = -(2)^2 + 4(2) + 10$
$s(2) = -4 + 8 + 10$
$s(2) = 14$

Vertex is (2, 14)
axis of symmetry is t = 2

Find y-intercept

$s(0) = -(0)^2 + 4(0) + 10$
$s(0) = 10$

y-intercept is at (0, 10)

Find x-intercept

$0 = -t^2 + 4t + 10$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(4) \pm \sqrt{(4)^2 - 4(-1)(10)}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{16 + 40}}{-2}$$

$$= \frac{4 \pm \sqrt{56}}{2}$$

$$= \frac{4 \pm 2\sqrt{14}}{2}$$

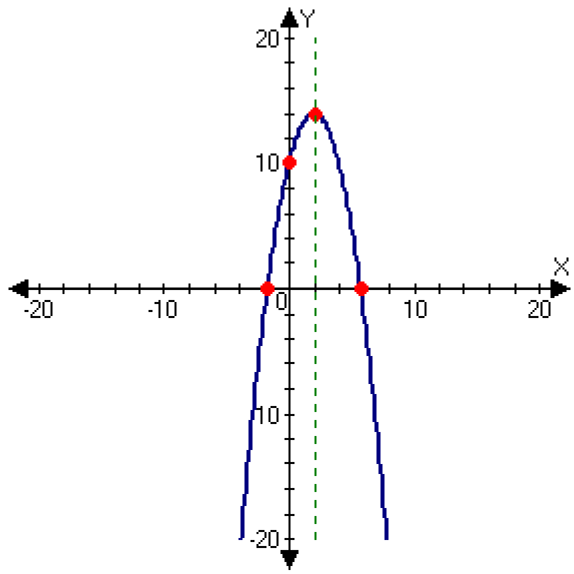
$$= 2 \pm \sqrt{14}$$

$$t = 2 + \sqrt{14} \quad or \quad t = 2 - \sqrt{14}$$

$$t = 5.74 \qquad t = -1.74$$

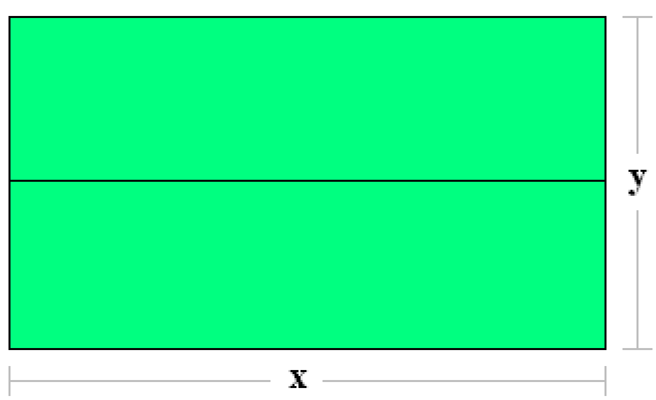
Exercise 4 (Continued):

Graph:



Exercise 5: A farmer has 1200 feet of fencing that he can use to enclose a rectangular lot and split it in two by another fence that runs parallel to one side of the lot. Find the dimensions of the lot that maximize the total enclosed area. What is the maximum area?

Draw image of the problem



Use the perimeter formula to express the dimensions in a single variable

$1200 = 3x + 2y$

$1200 - 3x = 2y$

$\frac{1200 - 3x}{2} = y$

$600 - \frac{3}{2}x = y$

Exercise 5 (Continued):

Use the area formula to setup the function

$A(x) = x(600 - \frac{3}{2}x)$

$A(x) = 600x - \frac{3}{2}x^2$

$A(x) = -\frac{3}{2}x^2 + 600x$

Locate the vertex

$$
\begin{aligned}
x &= \frac{-b}{2a} \\
&= -\frac{600}{2\left(-\frac{3}{2}\right)} \\
&= -\frac{600}{-3} \\
&= 200
\end{aligned}
$$

$A(200) = -\frac{3}{2}(200)^2 + 600(200)$

$A(200) = -\frac{3}{2}(40000) + 120000$

$A(200) = -60000 + 120000$

$A(200) = 60000$

The maximum area that can be enclosed is 60,000 square feet.