Review Exercise Set 9

Exercise 1: Determine if the given function is a polynomial function. If it is, then state its degree.

$$f(x) = 3x^6 - \pi x^4 + \sqrt{3}x^2 - 1$$

Exercise 2: Use the leading coefficient test to determine the end behavior of the graph of the given polynomial function.

$$h(x) = -5x^5 + 3x^3 - x + 4$$

Exercise 3: Find the zeros for the given polynomial function and give the multiplicity for each zero. Also, state whether the graph crosses or touches the x-axis at each zero.

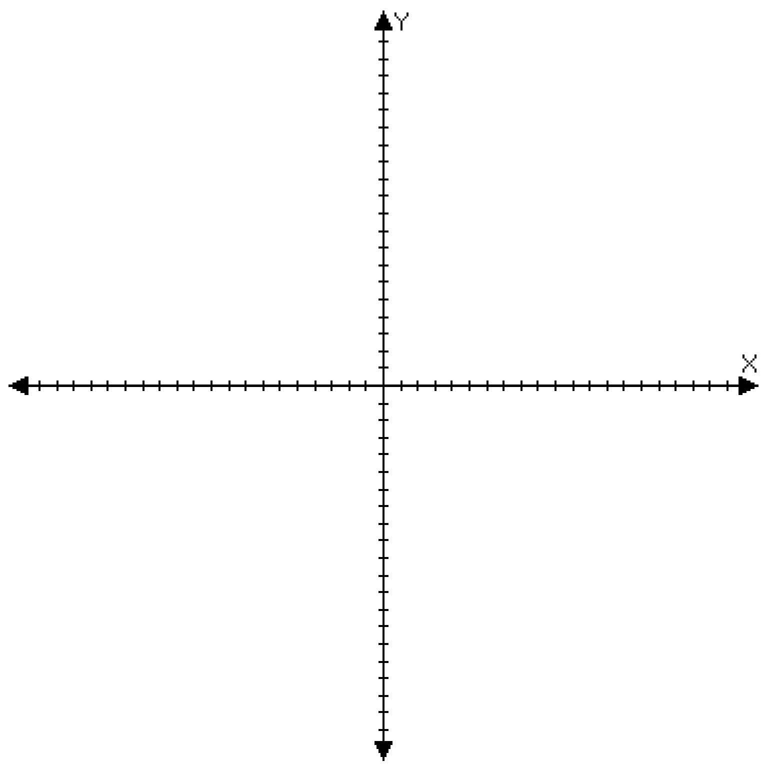
$$g(x) = -x(x - 2)^2(x + 1)^3$$

Exercise 4: Find the zeros for the given polynomial function and give the multiplicity for each zero. Also, state whether the graph crosses or touches the x-axis at each zero.

$$f(x) = -t^5 + 5t^4 + 4t^3 - 20t^2$$

Exercise 5: Graph the polynomial function $p(x) = x^2(x - 2)(x - 5)^3$ by finding the following information:

a) End behavior
b) Zeros
c) y-intercept
d) Symmetry (if any)
e) Additional points as needed



Review Exercise Set 9 Answer Key

Exercise 1: Determine if the given function is a polynomial function. If it is, then state its degree.

$$f(x) = 3x^6 - \pi x^4 + \sqrt{3}x^2 - 1$$

f(x) is a polynomial function because all of the coefficients are real numbers and the exponents are all nonnegative integers. The degree power of the polynomial is 6.

Exercise 2: Use the leading coefficient test to determine the end behavior of the graph of the given polynomial function.

$h(x) = -5x^5 + 3x^3 - x + 4$

The leading coefficient is negative and the degree of the polynomial is odd so the graph of the polynomial would increase without bound (rise) as x approaches negative infinity and decrease without bound (fall) as x approaches positive infinity.

Exercise 3: Find the zeros for the given polynomial function and give the multiplicity for each zero. Also, state whether the graph crosses or touches the x-axis at each zero.

$g(x) = -x(x - 2)^2(x + 1)^3$

Set the polynomial equal to zero

$0 = -x(x - 2)^2(x + 1)^3$

Set each factor equal to zero and solve for x

$-x = 0$
$x = 0$

$(x - 2)^2 = 0$
$x - 2 = 0$
$x = 2$

$(x + 1)^3 = 0$
$x + 1 = 0$
$x = -1$

Identify multiplicity and x-axis behavior

Zero	Multiplicity	x-axis behavior
x = 0	1	crosses
x = 2	2	touches
x = -1	3	crosses

Exercise 4: Find the zeros for the given polynomial function and give the multiplicity for each zero. Also, state whether the graph crosses or touches the x-axis at each zero.

$$f(x) = -x^5 + 5x^4 + 4x^3 - 20x^2$$

Set the polynomial equal to zero

$$0 = -x^5 + 5x^4 + 4x^3 - 20x^2$$

Factor the polynomial

$$0 = -x^2(x^3 - 5x^2 - 4x + 20)$$
$$0 = -x^2[(x^3 - 5x^2) + (-4x + 20)]$$
$$0 = -x^2[x^2(x - 5) - 4(x - 5)]$$
$$0 = -x^2(x^2 - 4)(x - 5)$$
$$0 = -x^2(x - 2)(x + 2)(x - 5)$$

Set each factor equal to zero and solve for t

$-x^2 = 0$	$x - 2 = 0$	$x + 2 = 0$	$x - 5 = 0$
$x = 0$	$x = 2$	$x = -2$	$x = 5$

Identify multiplicity and x-axis behavior

Zero	Multiplicity	x-axis behavior
x = 0	2	touches
x = 2	1	crosses
x = -2	1	crosses
x = 5	1	crosses

Exercise 5: Graph the polynomial function $p(x) = x^2(x - 2)(x - 5)^3$ by finding the following information:

a) End behavior

The degree of the polynomial function is the sum of the exponents which would give us a degree of 6. Since the coefficient of x in each factor is positive the leading coefficient of the polynomial will be positive. Therefore, with an even degree and a positive leading coefficient the function would increase without bound as x approaches both negative and positive infinity.

Exercise 5 (Continued):

b) Zeros

$p(x) = x^2(x - 2)(x - 5)^3$
$0 = x^2(x - 2)(x - 5)^3$

$x^2 = 0$	$x - 2 = 0$	$(x - 5)^3 = 0$
$x = 0$	$x = 2$	$x - 5 = 0$
		$x = 5$

The zeros of the polynomial are 0, 2, and 5.

c) y-intercept

$p(x) = x^2(x - 2)(x - 5)^3$
$p(0) = (0)^2(0 - 2)(0 - 5)^3$
$p(0) = (0)(-2)(-125)$
$p(0) = 0$

The y-intercept is at the origin (0, 0)

d) Symmetry (if any)

$p(-x) = (-x)^2(-x - 2)(-x - 5)^3$
$p(-x) = x^2(-1)(x + 2)[(-1)(x + 5)]^3$
$p(-x) = x^2(x + 2)(x + 5)^3$
$p(-x) \neq p(x)$ so the polynomial is not symmetry to the y-axis

$-p(x) = -x^2(x - 2)(x - 5)^3$
$p(-x) \neq -p(x)$ so the polynomial is not symmetry to the origin

e) Additional points as needed

$p(1) = (1)^2(1 - 2)(1 - 5)^3$	$p(3) = (3)^2(3 - 2)(3 - 5)^3$
$p(1) = 1(-1)(-4)^3$	$p(3) = 9(1)(-2)^3$
$p(1) = 64$	$p(3) = -72$

Exercise 5 (Continued):

Graph: