Review Exercise Set 24

Exercise 1: Write the first five terms of the sequence that has the given general term.

$$a_n = (-1)^{n-1} n^3$$

Exercise 2: Write the first five terms of the sequence that has the given recursion formula.

$$a_n = (2n - 1)\, a_{n-1} \,;\ a_1 = 2 \,;\ n \geq 2$$

Exercise 3: Evaluate the given factorial expression.

$$\frac{20!}{5!15!}$$

Exercise 4: Find the given sum.

$$\sum_{i=0}^{4} \frac{(-1)^{i+1} x^i}{i!}$$

Exercise 5: Express the given sum using summation notation.

1 - 4 + 9 - 16 + ... - 64

Review Exercise Set 24 Answer Key

Exercise 1: Write the first five terms of the sequence that has the given general term.

$a_n = (-1)^{n-1}\, n^3$

Substitute the values of 1 through 5 for n

$a_1 = (-1)^{1-1}\,(1)^3$
$a_1 = (-1)^0\,(1)$
$a_1 = (1)(1)$
$a_1 = 1$

$a_2 = (-1)^{2-1}\,(2)^3$
$a_2 = (-1)^1\,(8)$
$a_2 = (-1)(8)$
$a_2 = -8$

$a_3 = (-1)^{3-1}\,(3)^3$
$a_3 = (-1)^2\,(27)$
$a_3 = (1)(27)$
$a_3 = 27$

$a_4 = (-1)^{4-1}\,(4)^3$
$a_4 = (-1)^3\,(64)$
$a_4 = (-1)(64)$
$a_4 = -64$

$a_5 = (-1)^{5-1}\,(5)^3$
$a_5 = (-1)^4\,(125)$
$a_5 = (1)(125)$
$a_5 = 125$

The first five terms of the sequence are 1, -8, 27, -64, and 125.

Exercise 2: Write the first five terms of the sequence that has the given recursion formula.

$a_n = (2n - 1)\, a_{n-1}\ ;\ a_1 = 2\ ;\ n \geq 2$

Substitute the values of 2 through 5 for n since we already have the first term

$a_2 = (2(2) - 1)\, a_{2-1}$
$a_2 = (4 - 1)\, a_1$
$a_2 = (3)(2)$
$a_2 = 6$

$a_3 = (2(3) - 1)\, a_{3-1}$
$a_3 = (6 - 1)\, a_2$
$a_3 = (5)(6)$
$a_3 = 30$

$a_4 = (2(4) - 1)\, a_{4-1}$
$a_4 = (8 - 1)\, a_3$
$a_4 = (7)(30)$
$a_4 = 210$

$a_5 = (2(5) - 1)\, a_{5-1}$
$a_5 = (10 - 1)\, a_4$
$a_5 = (9)(210)$
$a_5 = 1890$

The first five terms of the sequence are 2, 6, 30, 210, and 1890.

Exercise 3: Evaluate the given factorial expression.

$$\frac{20!}{5!15!}$$

Expand the factorials

The 20! in the numerator will be expanded only until 15! since it can be reduced with the 15! in the denominator.

$$= \frac{20\times19\times18\times17\times16\times15!}{5\times4\times3\times2\times1\times15!}$$

$$= \frac{\not{20}\times19\times\not{18}^{3}\times17\times16\times\not{15!}}{\not{5}\times\not{4}\times\not{3}\times\not{2}\times1\times\not{15!}}$$

= 19 x 3 x 17 x 16
= 15,504

Exercise 4: Find the given sum.

$$\sum_{i=0}^{4}\frac{(-1)^{i+1}x^{i}}{i!}$$

Substitute in the values 0 through 4 for *i*

$$= \frac{(-1)^{0+1}x^{0}}{0!} + \frac{(-1)^{1+1}x^{1}}{1!} + \frac{(-1)^{2+1}x^{2}}{2!} + \frac{(-1)^{3+1}x^{3}}{3!} + \frac{(-1)^{4+1}x^{4}}{4!}$$

$$= \frac{(-1)^{1}(1)}{1} + \frac{(-1)^{2}x}{1} + \frac{(-1)^{3}x^{2}}{2} + \frac{(-1)^{4}x^{3}}{6} + \frac{(-1)^{5}x^{4}}{24}$$

$$= -1 + x - \frac{1}{2}x^{2} + \frac{1}{6}x^{3} - \frac{1}{24}x^{4}$$

Exercise 5: Express the given sum using summation notation.

1 - 4 + 9 - 16 + ... - 64

Rewrite each term in the series to determine a pattern

Each term in the series is a perfect square

$1^2 - 2^2 + 3^2 - 4^2 + ... - 8^2$

Exercise 5 (Continued):

The signs are alternating so there must be a (-1) in the pattern. Since the first term if positive the exponent would contain a (+ 1).

$$(-1)^{1+1}\,1^2 + (-1)^{2+1}\,2^2 + (-1)^{3+1}\,3^2 + (-1)^{4+1}\,4^2 + \ldots + (-1)^{8+1}\,8^2$$

Now replace the numbers that change between each term with i to obtain the pattern of the general term

$$(-1)^{i+1}\,i^2$$

Now write the sum using summation notation

$$1 - 4 + 9 - 16 + \ldots - 64 = \sum_{i=1}^{8} (-1)^{i+1} i^2$$