Review Exercise Set 24

Exercise 1: Write the first five terms of the sequence that has the given general term.

\[ a_n = (-1)^{n-1} n^3 \]

Exercise 2: Write the first five terms of the sequence that has the given recursion formula.

\[ a_n = (2n - 1) a_{n-1} ; a_1 = 2 ; n \geq 2 \]

Exercise 3: Evaluate the given factorial expression.

\[
\frac{20!}{5!15!}
\]
Exercise 4: Find the given sum.

\[ \sum_{j=0}^{4} \frac{(-1)^{j+1} x^j}{j!} \]

Exercise 5: Express the given sum using summation notation.

\[ 1 - 4 + 9 - 16 + \ldots - 64 \]
Review Exercise Set 24 Answer Key

Exercise 1: Write the first five terms of the sequence that has the given general term.

\[ a_n = (-1)^{n-1} n^3 \]

Substitute the values of 1 through 5 for \( n \)

\[
\begin{align*}
a_1 &= (-1)^{1-1} (1)^3 \\
a_2 &= (-1)^{2-1} (2)^3 \\
a_3 &= (-1)^{3-1} (3)^3 \\
a_4 &= (-1)^{4-1} (4)^3 \\
a_5 &= (-1)^{5-1} (5)^3
\end{align*}
\]

\[
\begin{align*}
a_1 &= (-1)^0 (1) \\
a_2 &= (-1)^1 (8) \\
a_3 &= (-1)^2 (27) \\
a_4 &= (-1)^3 (64) \\
a_5 &= (-1)^4 (125)
\end{align*}
\]

\[
\begin{align*}
a_1 &= 1 \\
a_2 &= -8 \\
a_3 &= 27 \\
a_4 &= -64 \\
a_5 &= 125
\end{align*}
\]

The first five terms of the sequence are 1, -8, 27, -64, and 125.

Exercise 2: Write the first five terms of the sequence that has the given recursion formula.

\[ a_n = (2n - 1) a_{n-1} ; a_1 = 2 ; n \geq 2 \]

Substitute the values of 2 through 5 for \( n \) since we already have the first term

\[
\begin{align*}
a_2 &= (2(2) - 1) a_1 \\
a_3 &= (2(3) - 1) a_2 \\
a_4 &= (2(4) - 1) a_3 \\
a_5 &= (2(5) - 1) a_4
\end{align*}
\]

\[
\begin{align*}
a_2 &= (4 - 1) a_1 \\
a_3 &= (6 - 1) a_2 \\
a_4 &= (8 - 1) a_3 \\
a_5 &= (10 - 1) a_4
\end{align*}
\]

\[
\begin{align*}
a_2 &= 6 \\
a_3 &= 30 \\
a_4 &= 210 \\
a_5 &= 1890
\end{align*}
\]

The first five terms of the sequence are 2, 6, 30, 210, and 1890.
Exercise 3: Evaluate the given factorial expression.

\[
\frac{20!}{5!15!}
\]

Expand the factorials

The 20! in the numerator will be expanded only until 15! since it can be reduced with the 15! in the denominator.

\[
\frac{20\times19\times18\times17\times15!}{5\times4\times3\times2\times1\times15!}
\]
\[
\frac{20\times19\times18\times17\times15!}{5\times4\times3\times2\times1\times15!}
\]
\[
= 19 \times 3 \times 17 \times 16
\]
\[
= 15,504
\]

Exercise 4: Find the given sum.

\[
\sum_{i=0}^{4} \frac{(-1)^{i+1} x^i}{i!}
\]

Substitute in the values 0 through 4 for \(i\)

\[
\frac{(-1)^{0+1} x^0}{0!} + \frac{(-1)^{1+1} x^1}{1!} + \frac{(-1)^{2+1} x^2}{2!} + \frac{(-1)^{3+1} x^3}{3!} + \frac{(-1)^{4+1} x^4}{4!}
\]
\[
\frac{(-1)^{1}(1)}{1} + \frac{(-1)^{2} x}{2} + \frac{(-1)^{3} x^2}{3} + \frac{(-1)^{4} x^3}{6} + \frac{(-1)^{5} x^4}{24}
\]
\[
= -1 + x - \frac{1}{2} x^2 + \frac{1}{6} x^3 - \frac{1}{24} x^4
\]

Exercise 5: Express the given sum using summation notation.

\[
1 - 4 + 9 - 16 + \ldots - 64
\]

Rewrite each term in the series to determine a pattern

Each term in the series is a perfect square

\[
1^2 - 2^2 + 3^2 - 4^2 + \ldots - 8^2
\]
Exercise 5 (Continued):

The signs are alternating so there must be a (-1) in the pattern. Since the first term if positive the exponent would contain a (+ 1).

\(-1)^{i+1} \cdot i^2 + (-1)^{2i+1} \cdot 2^2 + (-1)^{3i+1} \cdot 3^2 + (-1)^{4i+1} \cdot 4^2 + ... + (-1)^{8i+1} \cdot 8^2\)

Now replace the numbers that change between each term with \(i\) to obtain the pattern of the general term

\(-1)^{i+1} \cdot i^2\)

Now write the sum using summation notation

\[1 - 4 + 9 - 16 + ... - 64 = \sum_{i=1}^{8} (-1)^{i+1} \cdot i^2\]