Review Exercise Set 27

Exercise 1: Evaluate the given binomial coefficient.

\[
\binom{15}{7}
\]

Exercise 2: Expand the given expression using the binomial theorem.

\[(x + 2)^3\]

Exercise 3: Expand the given expression using the binomial theorem.

\[(3x - 4y)^5\]
Exercise 4: Find the indicated term of the binomial expansion for the given expression.

\[ 5^{\text{th}} \text{ term of } (2x - 5y)^8 \]

Exercise 5: Find the indicated term of the binomial expansion for the given expression.

\[ 8^{\text{th}} \text{ term of } (x + 3y)^{10} \]
Review Exercise Set 27 Answer Key

Exercise 1: Evaluate the given binomial coefficient.

\[
\binom{15}{7} = \frac{15!}{7!(15-7)!} = \frac{15!}{7!8!} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8!}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 8!} = 13 \times 11 \times 5 \times 9 = 6,435
\]

Exercise 2: Expand the given expression using the binomial theorem.

\[
(x + 2)^3 = \binom{3}{0}(x)^3 + \binom{3}{1}(x)^2(2)^1 + \binom{3}{2}(x)^1(2)^2 + \binom{3}{3}(2)^3
\]

\[
= \frac{3!}{0!3!}(x^3) + \frac{3!}{1!2!}(2x^2) + \frac{3!}{2!1!}(4x) + \frac{3!}{3!0!}(8)
\]

\[
= (1)(x^3) + (3)(2x^2) + (3)(4x) + (1)(8)
\]

\[
= x^3 + 6x^2 + 12x + 8
\]
Exercise 3: Expand the given expression using the binomial theorem.

\[(3x - 4y)^5\]

\[= \binom{5}{0}(3x)^5 + \binom{5}{1}(3x)^4(-4y)^1 + \binom{5}{2}(3x)^3(-4y)^2 + \binom{5}{3}(3x)^2(-4y)^3 + \binom{5}{4}(3x)^1(-4y)^4 + \binom{5}{5}(-4y)^5\]

\[= \frac{5!}{0!5!}(243x^5) + \frac{5!}{1!4!}(81x^4)(-4y) + \frac{5!}{2!3!}(27x^3)(16y^2) + \frac{5!}{3!2!}(9x^2)(-64y^3) + \frac{5!}{4!1!}(3x)(256y^4) + \frac{5!}{5!0!}(-1024y^5)\]

\[= (1)(243x^5) + (5)(-324x^4y) + (10)(432x^3y^2) + (10)(-576x^2y^3) + (5)(768xy^4) + (1)(-1024y^5)\]

\[= 243x^5 - 1620x^4y + 4320x^3y^2 - 5760x^2y^3 + 3840xy^4 - 1024y^5\]

Exercise 4: Find the indicated term of the binomial expansion for the given expression.

5th term of \((2x - 5y)^8\)

Find the value of \(r\)

\(r\) is always 1 more than the term so for the 5th term

\(r + 1 = 5\)

\(r = 4\)

Find the 5th term

\(n = 8; r = 4; a = 2x; b = -5y\)

\[\binom{n}{r}a^{n-r}b^r\]

\[= \binom{8}{4}(2x)^{8-4}(-5y)^4\]

\[= \frac{8!}{4!4!}(16x^4)(625y^4)\]

\[= (70)(10,000x^4y^4)\]

\[= 700,000x^4y^4\]
Exercise 5: Find the indicated term of the binomial expansion for the given expression.

8th term of \((x + 3y)^{10}\)

Find the value of \(r\)

For the 8th term

\[ r + 1 = 8 \]
\[ r = 7 \]

Find the 8th term

\[ n = 10; \ r = 7; \ a = x; \ b = 3y \]

\[
\begin{align*}
\binom{n}{r} a^{n-r} b^r \\
= \binom{10}{7} (x)^{10-7} (3y)^7 \\
= \frac{10!}{7!3!}(x^3)(2187y^7) \\
= (120)(2187x^3y^7) \\
= 262,440x^3y^7
\end{align*}
\]