The Algebra of Functions

Like terms, functions may be combined by addition, subtraction, multiplication or division.

Example 1. Given $f(x) = 2x + 1$ and $g(x) = x^2 + 2x - 1$ find $(f + g)(x)$ *and* $(f + g)(2)$

Solution

Step 1. **Find $(f + g)(x)$**

Since $(f + g)(x) = f(x) + g(x)$ then;

$$\begin{aligned}(f + g)(x) &= (2x + 1) + (x^2 + 2x - 1)\\ &= 2x + 1 + x^2 + 2x - 1\\ &= x^2 + 4x\end{aligned}$$

Step 2. **Find $(f + g)(2)$**

To find the solution for $(f + g)(2)$, evaluate the solution above for 2.

Since $(f + g)(x) = x^2 + 4x$ then;

$$\begin{aligned}(f + g)(2) &= 2^2 + 4(2)\\ &= 4 + 8\\ &= 12\end{aligned}$$

Example 2. Given $f(x) = 2x - 5$ and $g(x) = 1 - x$ find $(f - g)(x)$ and $(f - g)(2)$.

Solution

Step 1. **Find $(f - g)(x)$.**

$$\begin{aligned}(f - g)(x) &= f(x) - g(x)\\ &= (2x - 5) - (1 - x)\\ &= 2x - 5 - 1 + x\\ &= 3x - 6\end{aligned}$$

Step 2. **Find $(f - g)(2)$.**

$$\begin{aligned}(f - g)(x) &= 3x - 6\\ (f - g)(2) &= 3(2) - 6\\ &= 6 - 6\\ &= 0\end{aligned}$$

Example 3. Given $f(x) = x^2 + 1$ and $g(x) = x - 4$, find $(fg)(x)$ and $(fg)(3)$.

Solution

Step 1. Solve for $(fg)(x)$.

Since $(fg)(x) = f(x) * g(x)$, then

$$= (x^2 + 1)(x - 4)$$
$$= x^3 - 4x^2 + x - 4.$$

Step 2. Find $(fg)(3)$.

Since $(fg)(x) = x^3 - 4x^2 + x - 4$, then

$$(fg)(3) = (3)^3 - 4(3)^2 + (3) - 4$$
$$= 27 - 36 + 3 - 4$$
$$= -10$$

Example 4. Given $f(x) = x + 1$ and $g(x) = x - 1$, find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(3)$.

Solution

Step 1. Solve for $\left(\frac{f}{g}\right)(x)$.

Since $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, then

$$= \frac{x+1}{x-1}\ ;\ x \neq 1$$

Step 2 Find $\left(\frac{f}{g}\right)(3)$.

Since $\left(\frac{f}{g}\right)(x) = \frac{x+1}{x-1}$, then

$$\left(\frac{f}{g}\right)(3) = \frac{3+1}{3-1}$$
$$= \frac{4}{2}$$
$$= 2$$

NOTE: Any restrictions on the domains of *f* or *g* must be taken into account when performing these operations.
Composition is another operation that may be performed among functions. Simply stated, it is evaluating one function in terms of another. The format for composition is: $(f \circ g)(x) = f(g(x))$.

Example 5. Given $f(x) = x^2$ and $g(x) = x + 1$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

Solution

Step 1. Find $(f \circ g)(x)$

Since $(f \circ g)(x) = f(g(x))$, then
$= f(x + 1)$
$= (x + 1)^2$

Step 2. Find $(g \circ f)(x)$

Since $(g \circ f)(x) = g(f(x))$, then
$= g(x^2)$
$= (x^2) + 1$

Note that $(f \circ g)(x) \neq (g \circ f)(x)$. This means that, unlike multiplication or addition, composition of functions is not a commutative operation.

The following example will demonstrate how to evaluate a composition for a given value.

Example 6. Find $(f \circ g)(3)$ and $(g \circ f)(3)$ if $f(x) = x + 2$ and $g(x) = 4 - x^2$.

Solution

Step 1. Find $(f \circ g)(x)$ then evaluate for *3*.

Since $(f \circ g)(x) = f(g(x))$, then
$= f(4 - x^2)$
$= (4 - x^2) + 2$
$= 6 - x^2$

Evaluating for 3

$(f \circ g)(3) = 6 - (3)^2$
$= 6 - 9$
$= -3$

Example 6 (Continued):

Step 2. Find $(g \circ f)(x)$ then evaluate for 3.

Since $(g \circ f)(x) = g(\ f(x)\ )$, then

$$= g\,(x + 2)$$
$$= 4 - (x + 2)^2$$

Evaluating for 3

$$(g \circ f)(3) = 4 - (3 + 2)^2$$
$$= 4 - (5)^2$$
$$= 4 - 25$$
$$= -21$$